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ARTICLE *in* JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS · FEBRUARY 2015

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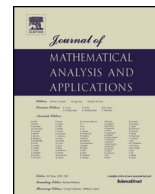


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Models for determining how many natural enemies to release inoculatively in combinations of biological and chemical control with pesticide resistance



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ARTICLE INFO

Article history:

Received 21 May 2014

Available online 7 October 2014

Submitted by J. Shi

Keywords:

IPM

Pest resistance

Biological control

Inoculative releases

Dynamic threshold

ABSTRACT

Combining biological and chemical control has been an efficient strategy to combat the evolution of pesticide resistance. Continuous releases of natural enemies could reduce the impact of a pesticide on them and the number to be released should be adapted to the development of pesticide resistance. To provide some insights towards this adaptation strategy, we developed a novel pest–natural enemy model considering both resistance development and inoculative releases of natural enemies. Three releasing functions which ensure the extinction of the pest population are proposed and their corresponding threshold conditions obtained. Aiming to eradicate the pest population, an analytic formula for the number of natural enemies to be released was obtained for each of the three different releasing functions, with emphasis on their biological implications. The results can assist in the design of appropriate control strategies and decision-making in pest management.

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1. Introduction

Chemical pest control is defined as the reduction of a pest population by using chemical pesticides. Because the latter are relatively cheap and are readily available, chemical pest control is the most common method used. However, with regular and repeated spraying, a pest may develop resistance to the pesticide quickly. As a consequence, there has been decreased susceptibility of pest populations to pesticides that were previously effective [23]. Studies indicate that more than 500 species of pests have now developed resistance to some pesticides [10,15,34]. Pesticide resistance also leads to increases in farmers' losses, even

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though more pesticides are used. For example, in the USA, farmers lost 7% of their crops to pests in the 1940s, but since the 1980s the percentage lost has increased to 13%; nevertheless, even more pesticides are still being used [35].

Therefore, in order to reduce or delay the development of pesticide resistance, a number of strategies have been proposed including pesticide switching or rotation, avoiding unnecessary pesticide applications, leaving untreated refuges where susceptible pests can survive, and using non-chemical control techniques [7]. The concept of integrated pest management (IPM) [9,21,38,36,37,24], an integrated combination of more than one method (such as biological control, chemical control, cultural tactics, breeding for host-plant resistance etc.), has been developed aiming to maintaining the density of pest populations below their Economic Injury Levels (EIL).

Pesticide switching or rotation is the main and usual method to fight pesticide resistance. In our recent study [16], we developed a pest population growth model incorporating the evolution of pesticide resistance, and introduced three different pesticide switching methods: threshold condition-guided, density-guided and EIL-guided. For each method, we discussed the optimal switching time. Moreover, we compared these three methods, and our results indicated that either the density-guided method or the EIL-guided method was the optimal pesticide switching method, depending on the frequency (or period) of pesticide applications.

Although pesticide switching is an efficient pest control method, it may initiate multi-pesticide resistance against which IPM is proposed. Biological control is often a key component of such an IPM strategy [11,22,27], and is the key method for responding to pesticide resistance [25]. Biological control aims to reduce pest populations by releasing natural enemies at some critical time when insufficient reproduction of released natural enemies is likely to occur and pest control will be achieved exclusively by the released individuals themselves [13,20]. This approach is known as augmentation of natural enemies. There are two general means to augmentation: inundative releases and inoculative releases [25].

Inundative release, the releasing of large numbers of natural enemies for immediate reduction of a damaging or near-damaging pest population, has been used frequently. Examples include the mass release of the egg parasite *Trichogramma* for controlling the eggs of various types of moths [25]. This approach is usually implemented impulsively, and it has been widely studied through mathematical models, especially with impulsive differential equations [20,29,30,28,31–33,17]. For example, Liang et al. [17] developed two impulsive pest–natural enemy interaction models with the development of pesticide resistance in which pulsed actions such as pesticide applications and natural enemy releases were considered. A goal of our present paper is to estimate how the number of natural enemies to be released should be changed with increasing pesticide resistance, to ensure pest eradication. Our analytic study shows how to change this according to the cumulative number of dead natural enemies before the next scheduled release time.

Inoculative releasing refers to the continuous releasing of small numbers of natural enemies at prescribed intervals throughout the pest period, starting when the pest population is very low. Examples include the release of predatory mites to protect greenhouse crops, and the inoculation of soils with the milky spore pathogen (*Bacillus popilliae*) to control Japanese beetle grubs [18,25]. This releasing strategy is also known as consecutive release.

So, if the pests develop resistance to the pesticide and biological control is implemented by inoculative release, how many natural enemies need to be released as the resistance to the pesticide evolves? To answer such questions, we develop a hybrid impulsive pest–natural enemy model with pesticide sprays in pulses, evolution of pesticide resistance and inoculative releases of natural enemies. Three different releasing methods are proposed and considered: (a) fixed numbers of natural enemies released over the period of pesticide application; (b) the numbers of natural enemies released is linearly dependent on time; (c) the numbers of natural enemies released is exponentially dependent on time. For each releasing method, we investigate the threshold condition for pest eradication, and the optimal number of natural enemies to be released at a particular time or at prescribed intervals.

2. Model and the threshold condition

2.1. IPM model with resistance to pesticides

In this section, we first introduce a simple pest population growth model concerning the evolution of pest resistance with impulsive pesticide spraying. We then propose an IPM model with evolution of pest resistance. Our main purpose is to investigate how to implement the IPM strategy (such as how to apply pesticide and how to release the natural enemies) with the development of the pest resistance such that the pest population dies out eventually.

In this study, we assume that the pest population follows the logistic growth equation, i.e.,

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K} \right), \tag{1}$$

where P denotes the size of the pest population, r represents the intrinsic growth rate and K represents the carrying capacity.

If pesticide resistance is taken into account, then the total pest population P should be divided into susceptible and resistant pests, denoted by P_s and P_r , respectively. In this case, the proportion of susceptible pests in the population is $\omega = P_s/P$, and then the proportion of resistant pests is $1 - \omega$. Therefore, the pest resistance to the pesticide can be described by $1 - \omega$ or by ω . Because the pest resistance is increasing as the number of pesticide applications increase, so ω is dynamically changed. To describe the dynamics of ω , Liang et al. proposed the following dynamic equation [16,17]

$$\frac{d\omega(t)}{dt} = d_1\omega(\omega^{q_i} - 1), \quad \tau_{i-1} \leq t \leq \tau_i, \quad i \in \mathcal{N}, \tag{2}$$

where q_i depends on the dosage and the frequency (or period) of pesticide applications. Thus, q_i is a function of the number of pesticide applications, the dosage X_i of the i -th pesticide application and the time interval $\Delta\tau_i = \tau_i - \tau_{i-1}$ between the i -th and $(i - 1)$ -th pesticide applications [19], and τ_{i-1} is the spraying time for $i \in \mathcal{N}$ with $\tau_0 = 0$, $\mathcal{N} = \{1, 2, 3, \dots\}$, and d_1 represents the mortality rate of susceptible pests due to the application of pesticide. By a simple calculation, the analytical solution of $\omega(t)$ can be determined as follows:

$$\omega(t) = \left(1 + e^{q_i d_1 (t - \tau_{i-1})} \left((\omega(\tau_{i-1}))^{-q_i} - 1 \right) \right)^{-\frac{1}{q_i}}, \quad \tau_{i-1} \leq t \leq \tau_i. \tag{3}$$

For convenience, in this work we assume that the pesticides are applied periodically, i.e. we have $\tau_i - \tau_{i-1} = T$, $i \in \mathcal{N}$. If so, we can define $q_i = i/T$ (i.e. the dosage of pesticide application is normalized as one), and then we have

$$\omega(nT) = \left(1 + e^{n d_1} \left(\omega((n - 1)T)^{-n/T} - 1 \right) \right)^{-T/n}, \quad n \in \mathcal{N}. \tag{4}$$

Although chemical control is one of the main control tactics of pest control, it may result in high rates of failure due to fast development of pest resistance to the pesticide. As mentioned in the introduction, combining chemical sprays with biological control is one of the main methods of weakening or delaying the resistance of pest to the pesticide.

Therefore, in order to depict the effects of pesticide resistance development on pest control in more detail, IPM strategies should be considered. That is, the growth of the natural enemy population needs to be included in the model. So, we have the following model:

$$\begin{cases} \frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right) - \alpha P(t)N(t), & t \neq nT, \\ \frac{dN(t)}{dt} = \beta\alpha P(t)N(t) - dN(t) + \delta(t), \\ P(nT^+) = (1 - \omega(nT)d_1)P(nT), & t = nT, \\ \frac{d\omega(t)}{dt} = d_1\omega(t)(\omega(t)^{q_n} - 1) \end{cases} \tag{5}$$

with initial value $P(0^+) = P_0$, $N(0^+) = N_0$, $\omega(0) = \omega_0$. Where $q_n = n/T$, and $N(t)$ is the population size of the natural enemy at time t , α denotes the attack rate of the predator, β represents conversion efficiency, d is the mortality rate of the natural enemy and $\delta(t)$ is the number of natural enemies released at time t .

As pest resistance to the pesticide develops, the effect of the pesticide wears off naturally. Thus, in order to control the density of the pest population below some critical level, farmers should increase the number of natural enemies to be released. In general, with the consideration of pesticide resistance development, it is normally difficult to depict the numbers of natural enemies to be released over time. In order to address this difficulty, we propose the following possible functions (termed releasing functions):

(i) *Step function*

$$\delta(t) = \delta_n, \quad t \in [nT, (n + 1)T), \tag{6}$$

where $\delta_n \geq 0$. In this case, we assume that a constant number of natural enemies is released during each pesticide application period T .

(ii) *Linear function*

$$\delta(t) = \theta t, \tag{7}$$

where θ is a non-negative constant. In this case, we assume that the number of natural enemies to be released depends on the time linearly. The difficulty in this case is how to determine the relations between the constant θ and time t , i.e. what is the limitation of function $\delta(t)$ as t approaches infinity?

(iii) *Exponential function*

$$\delta(t) = \exp(at), \tag{8}$$

where a is positive constant. In this case, we assume that the number of natural enemies to be released depends on time exponentially, and the same question as in case (ii) should be addressed.

Therefore, our main purposes are to determine the effects of the above three releasing functions on control of the pest, and then to discuss their biological implications.

2.2. Threshold condition

Here we consider model (5) and examine how to design the releasing functions $\delta(t)$ as pesticide resistance develops. Of particular interest is to determine the releasing functions $\delta(t)$ for a fixed period T so that the pest population dies out eventually without switching pesticides.

The basic properties of the following subsystem

$$\begin{cases} \frac{dN(t)}{dt} = -dN(t) + \delta(t), \\ N(0^+) = N_0 \end{cases} \tag{9}$$

play key roles for the investigation of model (5).

The analytical solution of this subsystem at any impulsive interval $((n - 1)T, nT]$ gives

$$N^*(t) = e^{-dt} \left(N_0 + \int_0^t \delta(s)e^{ds} ds \right). \tag{10}$$

Note that any two solutions of subsystem (9) have the same limit as $t \rightarrow \infty$.

Therefore, the expression of the pest-free solution of system (5) is given by

$$(0, N^*(t)) = \left(0, e^{-dt} \left(N_0 + \int_0^t \delta(s)e^{ds} ds \right) \right). \tag{11}$$

For $0 \leq l < T$ and $n \in \mathcal{N}$, we denote

$$G(n, l) = \left(-\alpha \int_{l+(n-1)T}^{l+nT} N^*(s) ds \right)$$

and

$$R_0(n, T) = (1 - d_1\omega(nT))e^{rT}.$$

Then we have the following threshold theorem for the global attractivity of the pest-free solution.

Theorem 2.1. *Let*

$$R_0^N(n, T, l) = R_0(n, T) \exp(G(n, l)) \tag{12}$$

for $0 \leq l < T$, and $(P(t), N(t))$ be any solution of system (5). Then the pest-free solution (11) is globally attractive if $R_0^N(n, T, l) \leq 1$.

The proof of this theorem is provided in Appendix A. Based on the main results shown in Theorem 2.1, we can now provide the formula of threshold condition $R_0^N(n, T, l)$ for three different releasing functions.

Case 2.2.1. Constant step releasing function, i.e. $\delta(t) = \delta_n, t \in [nT, (n + 1)T)$.

Substituting $\delta(t) = \delta_n, t \in [nT, (n + 1)T)$ into (9), we have

$$N^*(t) = \frac{\delta_n}{d} + \left(N(nT) - \frac{\delta_n}{d} \right) e^{-d(t-nT)}, \quad t \in [nT, (n + 1)T), \tag{13}$$

where

$$N(nT) = \frac{1 - e^{-dT}}{d} \sum_{i=0}^{n-1} \delta_i e^{-(n-1-i)dT} + N_0 e^{-dnT}.$$

Thus,

$$\begin{aligned} \int_{(n-1)T+l}^{nT+l} N^*(t)dt &= \int_{(n-1)T+l}^{nT} N^*(t)dt + \int_{nT}^{nT+l} N^*(t)dt \\ &= \int_{(n-1)T+l}^{nT} \left[\frac{\delta_{n-1}}{d} + \left(N((n-1)T) - \frac{\delta_{n-1}}{d} \right) e^{-d(t-(n-1)T)} \right] dt \\ &\quad + \int_{nT}^{nT+l} \left[\frac{\delta_n}{d} + \left(N(nT) - \frac{\delta_n}{d} \right) e^{-d(t-nT)} \right] dt \\ &= \left[\left(l - \frac{1 - e^{-dl}}{d} \right) \frac{\delta_n - \delta_{n-1}}{d} + \frac{e^{-dl}(1 - e^{-dT})}{d} \left(N((n-1)T) - \frac{\delta_{n-1}}{d} \right) + \frac{T}{d} \delta_{n-1} \right], \end{aligned}$$

therefore,

$$\begin{aligned} G(n, l) &= -\alpha \int_{(n-1)T+l}^{nT+l} N^*(t)dt \\ &= -\frac{\alpha}{d} \left[\left(l - \frac{1 - e^{-dl}}{d} \right) (\delta_n - \delta_{n-1}) + e^{-dl}(1 - e^{-dT}) \left(N((n-1)T) - \frac{\delta_{n-1}}{d} \right) + T\delta_{n-1} \right] \\ &\doteq G_{\delta_n}(n, l), \end{aligned} \tag{14}$$

and the threshold value is given by $R_0^N(n, T, l) = R_0(n, T) \exp(G_{\delta_n}(n, l)) \doteq R_{0\delta_n}^N(n, T, l)$.

In particular, if $\delta(t) = \delta$ for all $t \in [0, +\infty)$, then

$$N^*(t) = \frac{\delta}{d} + \left(N_0 - \frac{\delta}{d} \right) e^{-dt} \doteq N_{\delta}^*(t),$$

thus

$$\begin{aligned} G(n, l) &= -\alpha \int_{(n-1)T+l}^{nT+l} N^*(t)dt \\ &= -\frac{\alpha}{d} \left[\delta T + \left(N_0 - \frac{\delta}{d} \right) e^{-d(l+(n-1)T)} (1 - e^{-dT}) \right] \\ &\doteq G_{\delta}(n, l), \end{aligned} \tag{15}$$

and the threshold value is $R_0^N(n, T, l) = R_0(n, T) \exp(G_{\delta}(n, l)) \doteq R_{0\delta}^N(n, T, l)$.

Case 2.2.2. Linear releasing function i.e. $\delta(t) = \theta t$ for all $t \in [0, +\infty)$.

In this case,

$$N^*(t) = \frac{\theta}{d} \left(t - \frac{1}{d} \right) + \left(N_0 + \frac{\theta}{d^2} \right) e^{-dt} \doteq N_{\theta}^*(t),$$

thus,

$$\begin{aligned}
 G(n, l) &= -\alpha \int_{(n-1)T+l}^{nT+l} N^*(t) dt \\
 &= -\frac{\alpha}{d} \left[\theta T \left(\frac{(2n-1)T}{2} + l - \frac{1}{d} \right) - \left(N_0 + \frac{\theta}{d^2} \right) e^{-d(l+(n-1)T)} (e^{-dT} - 1) \right] \\
 &\doteq G_\theta(n, l),
 \end{aligned} \tag{16}$$

and $R_0^N(n, T, l) = R_0(n, T) \exp(G_\theta(n, l)) \doteq R_{0\theta}^N(n, T, l)$.

Case 2.2.3. Exponential releasing function i.e. $\delta(t) = \exp(-at)$ for all $t \in [0, +\infty)$.

In this case, we have

$$N^*(t) = \left(N_0 - \frac{1}{d+a} \right) e^{-dt} + \frac{e^{at}}{d+a} \doteq N_a^*(t),$$

therefore,

$$\begin{aligned}
 G(n, l) &= -\alpha \int_{(n-1)T+l}^{nT+l} N^*(t) dt \\
 &= -\alpha \left[\frac{1}{d} \left(N_0 + \frac{1}{d+a} \right) e^{-d(l+(n-1)T)} (1 - e^{-dT}) + \frac{1}{a(d+a)} e^{a(l+(n-1)T)} (e^{aT} - 1) \right] \\
 &\doteq G_a(n, l),
 \end{aligned} \tag{17}$$

and $R_0^N(n, T, l) = R_0(n, T) \exp(G_a(n, l)) \doteq R_{0a}^N(n, T, l)$.

3. Releasing functions based on threshold condition

A main purpose of pest control is to completely eradicate the pest or maintain its density below the EIL. From the mathematical point of view, this means that we should keep the threshold value $R_0^N(n, T, l)$ below unity ([Theorem 2.1](#)) or a given level which is less than 1. From the above discussion, $R_0^N(n, T, l)$ consists of two parts: $R_0(n, T)$ (chemical control) and $\exp(G(n, l))$ (biological control). From (4), we conclude that $\omega(nT)$ is a decreasing function with respect to the number of increasing pesticide sprays, and this leads to the observation that the threshold value $R_0(n, T)$ is an increasing function with respect to the number of pesticide sprays. This also indicates that $R_0(n, T)$ could exceed 1 after several pesticide applications. Therefore, if only the chemical control is implemented, then pest resurgence could occur due to repeated use of the same type of pesticides and evolution of resistance to them. So in order to control the density of the pest population below the given level, farmers should increase the number of natural enemies released as the pest resistance to the pesticide evolves. But the question arises of how to determine the number of natural enemies to be released at any given time with the development of the pest’s resistance?

According to the above discussion, in this subsection we focus on determining the number of natural enemies to be released such that the threshold value $R_0^N(n, T, l)$ is less than a given level. That is how to determine δ_n , δ or θ in $R_{0\delta_n}^N(n, T, l)$, $R_{0\delta}^N(n, T, l)$ or $R_{0\theta}^N(n, T, l)$ such that those threshold values are always less than one. Without loss of generality, we assume those threshold values are less than a given constant R_C with $R_C \leq 1$. If so, the pest population can be successfully controlled and will die out eventually.

Let

$$R_{0_{\max}}^N(n, T, l) = \max_{0 \leq l < T} \{ R_0^N(n, T, l) \}, \quad G_{\max}(n, l) = \max_{0 \leq l < T} \{ G(n, l) \}.$$

Then

$$R_{0_{\max}}^N(n, T, l) \leq 1 \quad \text{implies} \quad R_0^N(n, T, l) \leq 1$$

for all $l \in [0, T)$. Therefore, in order to make sure $R_0^N(n, T, l) \leq 1$ for all $l \in [0, T)$, we only need $R_{0_{\max}}^N(n, T, l) = R_C \leq 1$.

In fact, we have

$$\frac{\partial R_0^N(n, T, l)}{\partial l} = R_0^N(n, T, l) \frac{\partial G(n, l)}{\partial l},$$

that is $R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\max}(n, l))$.

3.1. Determining the constant function δ

Due to the complexity of $R_{0_{\delta n}}^N(n, T, l)$, we let δ_n be a constant δ and consider the special case, i.e. we focus on $R_{0_{\delta}}^N(n, T, l)$ first.

It follows from the function $G_{\delta}(n, l)$ that

$$\frac{\partial G_{\delta}(n, l)}{\partial l} = \alpha \left(N_0 - \frac{\delta}{d} \right) e^{-d(l+(n-1)T)} (1 - e^{-dT}),$$

which indicates that if $N_0 > \delta/d$ then $G_{\delta}(n, l)$ is an increasing function with respect to l . Thus, we have

$$\begin{aligned} R_{0_{\delta_{\max}}}^N(n, T, l) &= R_0(n, T) \exp(G_{\delta}(n, T)) \\ &= R_0(n, T) \exp\left(-\frac{\alpha}{d} \left(\delta T + \left(N_0 - \frac{\delta}{d} \right) e^{-dnT} (1 - e^{-dT}) \right)\right) \\ &= R_{0_{\delta}}^N(n, T, T). \end{aligned}$$

Similarly, if $N_0 \leq \delta/d$ then $G_{\delta}(n, l)$ is a decreasing function with respect to l . Therefore, we have

$$\begin{aligned} R_{0_{\delta_{\max}}}^N(n, T, l) &= R_0(n, T) \exp(G_{\delta}(n, 0)) \\ &= R_0(n, T) \exp\left(-\frac{\alpha}{d} \left(\delta T + \left(N_0 - \frac{\delta}{d} \right) e^{-d(n-1)T} (1 - e^{-dT}) \right)\right) \\ &= R_{0_{\delta}}^N(n, T, 0). \end{aligned}$$

Based on the above, we consider the following two cases:

Case 3.1.1. $N_0 > \delta/d$.

In this case, solving the following equation

$$R_{0_{\delta_{\max}}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta}(n, T)) = R_C \tag{18}$$

with respect to δ , yields

$$\begin{aligned} \delta &= \frac{-\frac{d}{\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right) - N_0 e^{-dnT} (1 - e^{-dT})}{T - \frac{1}{d} e^{-dnT} (1 - e^{-dT})} \\ &\doteq \delta^1(n). \end{aligned} \tag{19}$$

It follows from $G_\delta(n, T) \leq 0$ and (18) that if $R_0(n, T) \leq R_C$ for some $n \in \mathcal{N}$, then $R_{0\delta}^N(n, T, l) \leq R_C$, which means that the chemical control alone can suppress the pest outbreak at the initial stage. However, once the pest resistance develops such that $R_0(n, T) > R_C$, then the releases of natural enemies are necessary to maintain $R_{0\delta}^N(n, T, T)$ as a constant R_C . All these results confirm that δ strictly depends on the number of pesticide applications n . Therefore, the number of natural enemies to be released (i.e., δ for all $n \in \mathcal{N}$) can be defined as follows

$$\delta = \begin{cases} \delta_c, & \text{if } R_0(n, T) \leq R_C, \\ \delta^1(n), & \text{if } R_0(n, T) > R_C, \end{cases} \tag{20}$$

where δ_c can be zero or a relatively small positive constant.

Due to the evolution of pest resistance we can easily obtain the following limitation

$$R_0(n, T) = (1 - d_1\omega(nT))e^{rT} \rightarrow e^{rT} \quad \text{as } n \rightarrow \infty.$$

Therefore,

$$\delta^1(n) \rightarrow -\frac{d}{\alpha T} \ln R_C + \frac{dr}{\alpha} \quad \text{as } n \rightarrow \infty.$$

Case 3.1.2. $N_0 \leq \delta/d$.

In this case, we let

$$R_{0\delta_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_\delta(n, 0)) = R_C. \tag{21}$$

By using the same methods as those in Case 3.1.1 we can see that if $R_0(n, T) \leq R_C$ for some $n \in \mathcal{N}$ then we can let $\delta = \delta_C$; if $R_0(n, T) > R_C$ then solving Eq. (21) with respect to δ , yields

$$\begin{aligned} \delta &= \frac{-\frac{d}{\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right) - N_0 e^{-d(n-1)T} (1 - e^{-dT})}{T - \frac{1}{d} e^{-d(n-1)T} (1 - e^{-dT})} \\ &\doteq \delta^2(n). \end{aligned} \tag{22}$$

Therefore, the number of natural enemies to be released (i.e., δ for all $n \in \mathcal{N}$) can be defined as follows:

$$\delta = \begin{cases} \delta_c, & \text{if } R_0(n, T) \leq R_C, \\ \delta^2(n), & \text{if } R_0(n, T) > R_C \end{cases} \tag{23}$$

with

$$\delta^2(n) \rightarrow -\frac{d}{\alpha T} \ln R_C + \frac{dr}{\alpha} \quad \text{as } n \rightarrow \infty.$$

In the following, we adopt a numerical approach to analyze the effects of the inoculation releasing constant δ defined by formula (20) or (23) on the success of the pest control with the development of pesticide resistance. Fig. 1(a) plots the $R_{0\delta}^N$ with respect to n under constant $\delta = 0.1$, we can see that the threshold value $R_{0\delta}^N$ is increasing with respect to n if we fixed all parameters. Note that $R_{0\delta}^N$ will exceed one after applying three rounds of IPM strategies. In this strategy, the density of the pest population will decrease firstly due to the high efficacy of the pesticide at the initial stage and then the pest population will increase and even outbreak again after applying IPM strategies several times (see Fig. 1(b)), which means that a constant releasing strategy cannot be effective for long term pest management. Therefore, the number of natural enemies to be released should be changed with the evolution of pesticide resistance. Fig. 1(c)

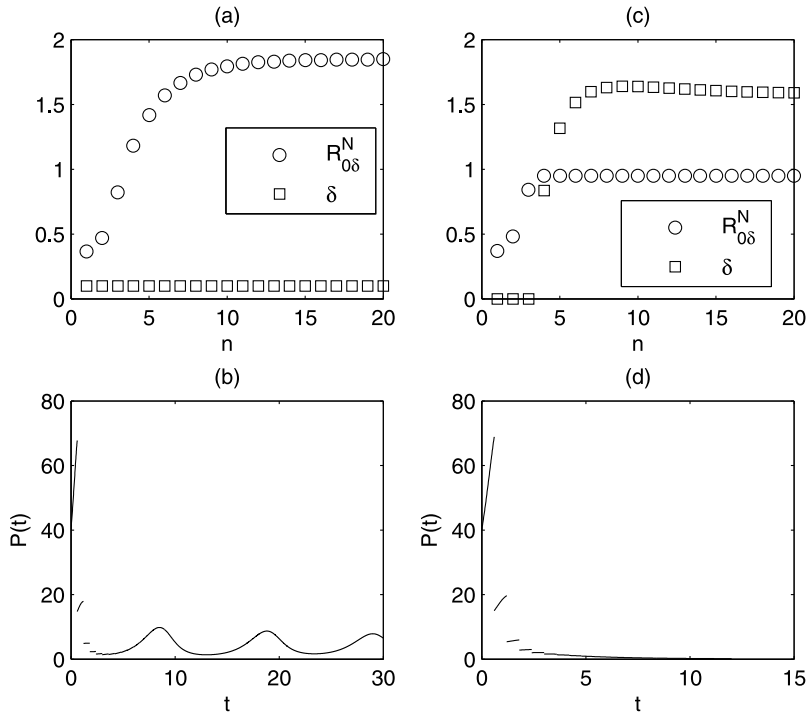


Fig. 1. Calculation of $R_{0\delta}^N$ and numerical simulations of model (5) with constant releasing of natural enemies. The baseline parameter values are as follows: $d_1 = 0.8$; $r = 1.1$; $d = 0.4$; $\alpha = 0.3$; $\omega_0 = 0.99$, $\beta = 0.3$, $T = 0.6$, $R_C = 0.95$, $\delta_c = 0$. (a) The plot of $R_{0\delta}^N$ with respect to n under constant $\delta = 0.1$ releases; (b) The time series of the pest population associated with (a); (c) The plot of $R_{0\delta}^N(n, T, l)$ with respect to n using the formula (20) to determine the δ ; (d) The time series of the pest population associated with (c).

plots $R_{0\delta}^N$ and δ with respect to n under $\delta(t) = \delta$ defined by formula (20). In this strategy, the pest population will die out after several rounds of pest control (see Fig. 1(d)).

3.2. Determining the step function δ_n

Now let us turn to the general case, i.e. the threshold value $R_{0\delta_n}^N(n, T, l)$.

Firstly, taking the derivative of the function $G_{\delta_n}(n, l)$ with respect to l yields:

$$\frac{\partial G_{\delta_n}(n, l)}{\partial l} = -\alpha \left[\frac{\delta_n - \delta_{n-1}}{d} - e^{-dl} \left(\left(N((n-1)T) - \frac{\delta_{n-1}}{d} \right) (1 - e^{-dT}) + \frac{\delta_n - \delta_{n-1}}{d} \right) \right], \quad (24)$$

where

$$N((n-1)T) = \frac{1 - e^{-dT}}{d} \sum_{i=0}^{n-2} \delta_i e^{-(n-2-i)dT} + N_0 e^{-d(n-1)T}.$$

In practice, in order to maintain the density of the pest population below the threshold value, the number of natural enemies to be released (i.e. δ_n) should be an increasing function with respect to n as the pest resistance develops and the pesticide efficiency decreases. This indicates that we must have $\delta_n \geq \delta_{n-1}$ for some $n \in \mathcal{N}$ in the initial stage. However, the number of natural enemies to be released could be reduced once the cumulative number of natural enemies reaches a certain level and the natural balance between predator and prey is restored. Based on the above and without loss of generality, we assume that there exists a positive constant h such that $\delta_n \geq \delta_{n-1}$ for $n \leq h$ and $\delta_n < \delta_{n-1}$ for $n > h$.

According to the expression of (24), for the first few pesticide applications (i.e. $\delta_n \geq \delta_{n-1}$ for $n \leq h$), if $N((n - 1)T) \leq \delta_{n-1}/d$, then we have $\partial G_{\delta_n}(n, l)/\partial l \leq 0$. This indicates that $G_{\delta_n}(n, l)$ is a decreasing function with respect to l , thus

$$R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta_{n_{\max}}}(n, l)) = R_0(n, T) \exp(G_{\delta_n}(n, 0)).$$

However, if $N((n - 1)T) > \delta_{n-1}/d$ and $\delta_n \geq \delta_{n-1}$ in the initial stage, then there is an l^* such that $\partial G_{\delta_n}(n, l^*)/\partial l = 0$. This shows that $G_{\delta_n}(n, l)$ reaches its maximal value at $l = l^*$, i.e., we have

$$R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta_{n_{\max}}}(n, l)) = R_0(n, T) \exp(G_{\delta_n}(n, l^*)),$$

where

$$l^* = \frac{1}{d} \ln \left(1 + \frac{(dN((n - 1)T) - \delta_{n-1})(1 - e^{-dT})}{\delta_n - \delta_{n-1}} \right). \tag{25}$$

After some pesticide applications, the number of natural enemies to be released is reduced such that $\delta_n < \delta_{n-1}$ for $n > h$, and if $N((n - 1)T) \leq \delta_{n-1}/d$, then

$$R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta_{n_{\max}}}(n, l)) = R_0(n, T) \exp(G_{\delta_n}(n, l^*)).$$

If $\delta_n < \delta_{n-1}$ and $N((n - 1)T) > \delta_{n-1}/d$, then we have $\partial G_{\delta_n}(n, l)/\partial l > 0$. This shows that $G_{\delta_n}(n, l)$ is an increasing function with respect to l . Thus

$$R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta_{n_{\max}}}(n, l)) = R_0(n, T) \exp(G_{\delta_n}(n, T)).$$

By employing the same ideas as for the threshold value $R_{0_{\delta}}^N(n, T, l)$ and the same methods as above, we assume, without loss of generality, that there exists an integer $n' \in \mathcal{N}^*$ ($\mathcal{N}^* = \{0, 1, 2, \dots\}$) such that (i) $R_0(n, T) \leq R_C$ for $n \leq n'$ due to the high effectiveness of pesticide applications in the initial stage and (ii) $R_0(n, T) > R_C$ for $n > n'$. Thus, we let $\delta_n = \delta_c$ for $n \leq n'$ and let $R_{0_{\delta_{n_{\max}}}}^N(n, T, l) = R_C$ for $n > n'$. In the light of the above, we consider the following two cases:

Case 3.2.1. $\delta_n \geq \delta_{n-1}$ (i.e. $n' < n \leq h$).

In this case, if $N((n - 1)T) \leq \delta_{n-1}/d$, then $G_{\delta_n}(n, 0) = \ln(R_C/R_0(n, T))$, that is,

$$-\frac{\alpha}{d} \left(\delta_{n-1} \left(T - \frac{1 - e^{-dT}}{d} \right) + N((n - 1)T)(1 - e^{-dT}) \right) = \ln \left(\frac{R_C}{R_0(n, T)} \right).$$

Therefore,

$$\begin{aligned} \delta_n &= \frac{-\frac{d}{\alpha} \ln \left(\frac{R_C}{R_0(n+1, T)} \right) - \left(\frac{1 - e^{-dT}}{d} \sum_{i=0}^{n-1} \delta_i e^{-(n-1-i)dT} + N_0 e^{-dnT} \right) (1 - e^{-dT})}{T - \frac{1 - e^{-dT}}{d}} \\ &\doteq \delta_n^{(11)}. \end{aligned} \tag{26}$$

If $N((n - 1)T) > \delta_{n-1}/d$, then $R_{0_{\delta_{n_{\max}}}}^N(n, T, l) = R_{0_{\delta_n}}^N(n, T, l^*)$. Letting $R_{0_{\delta_{n_{\max}}}}^N(n, T, l) = R_C$, it follows from (12) that we have $G_{\delta_n}(n, l^*) = \ln(R_C/R_0(n, T))$, that is,

$$-\frac{\alpha}{d} (l^*(\delta_n - \delta_{n-1}) + T\delta_{n-1}) = \ln \left(\frac{R_C}{R_0(n, T)} \right), \tag{27}$$

where l^* is given by (25).

According to (27), we can determine δ_n for this case, we denote it as $\delta_n^{(12)}$.

Case 3.2.2. $\delta_n < \delta_{n-1}$ (i.e. $n > h$).

In this case, if $N((n - 1)T) \leq \delta_{n-1}/d$, then $R_{0\delta_{n_{\max}}}^N(n, T, l) = R_{0\delta_n}^N(n, T, l^*) = R_C$. It follows from (27) that we can determine δ_n as $\delta_n^{(12)}$.

If $N((n - 1)T) > \delta_{n-1}/d$, then $\partial G_{\delta_n}(n, l)/\partial l > 0$, that is, $G_{\delta_n}(n, l)$ is an increasing function with respect to l . So

$$R_{0\delta_{\max}}^N(n, T, l) = R_0(n, T) \exp(G_{\delta_{n_{\max}}}(n, l)) = R_0(n, T) \exp(G_{\delta_n}(n, T)).$$

Therefore, $R_{0\delta_{n_{\max}}}^N(n, T, l) = R_{0\delta_n}^N(n, T, T) = R_C$ indicate that $G_{\delta_n}(n, T) = \ln(R_C/R_0(n, T))$. Solving this equation with respect to δ_n , we have

$$\begin{aligned} \delta_n &= \frac{-\frac{d}{\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right) - (N((n - 1)T) - \frac{\delta_{n-1}}{d})(1 - e^{-dT})e^{-dT} - T\delta_{n-1}}{T - \frac{1 - e^{-dT}}{d}} + \delta_{n-1} \\ &= \frac{-\frac{d}{\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right) - \left(\frac{1 - e^{-dT}}{d} \sum_{i=0}^{n-2} \delta_i e^{-(n-2-i)dT} + N_0 e^{-d(n-1)T} - \frac{\delta_{n-1}}{d}\right)(1 - e^{-dT})e^{-dT}}{T - \frac{1 - e^{-dT}}{d}} \\ &\quad - \frac{T\delta_{n-1}}{T - \frac{1 - e^{-dT}}{d}} + \delta_{n-1} \\ &\doteq \delta_n^{(21)}. \end{aligned} \tag{28}$$

In summary, we can see that δ_n for all $n \in \mathcal{N}^*$ can be defined as follows:

$$\delta_n = \begin{cases} \delta_c, & \text{if } n \leq n', \\ \delta_n^{(11)}, & \text{if } n' < n \leq h \text{ and } N((n - 1)T) \leq \frac{\delta_{n-1}}{d}, \\ \delta_n^{(12)}, & \text{if } n' < n \leq h \text{ and } N((n - 1)T) > \frac{\delta_{n-1}}{d}, \\ \delta_n^{(12)}, & \text{if } n > h \text{ and } N((n - 1)T) \leq \frac{\delta_{n-1}}{d}, \\ \delta_n^{(21)}, & \text{if } n > h \text{ and } N((n - 1)T) > \frac{\delta_{n-1}}{d}. \end{cases} \tag{29}$$

Since $0 \leq \delta_n \leq \delta_{n-1}$ for $n > h$, we have $\delta_n - \delta_{n-1} \rightarrow 0$ as $n \rightarrow \infty$. It follows from $0 \leq l^* < T$ that

$$0 \geq l^*(\delta_n - \delta_{n-1}) > T(\delta_n - \delta_{n-1}) \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Thus, according to (27), we get

$$\delta_n^{(12)} \rightarrow -\frac{d}{\alpha T} \ln R_C + \frac{dr}{\alpha} \quad \text{as } n \rightarrow \infty.$$

Further, according to (13), we can get

$$N((n - 1)T) - \delta_{n-1} = \frac{\delta_{n-2} - \delta_{n-1}}{d} + (N((n - 2)T) - \delta_{n-2})e^{-dT},$$

which indicates that

$$N((n - 1)T) - \delta_{n-1} \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Therefore, according to (28), we have

$$\delta_n^{(21)} \rightarrow -\frac{d}{\alpha T} \ln R_C + \frac{dr}{\alpha} \quad \text{as } n \rightarrow \infty.$$

3.3. Determining the linear function θt

In this subsection, we will investigate how to determine θ (that is how to determine the number of natural enemies θt to be released at time t) based on the threshold value $R_{0\theta}^N(n, T, l)$.

By using the same methods as those shown in Subsection 3.2, differentiating $G_\theta(n, l)$ with respect to l we can get

$$\begin{aligned} \frac{\partial G_\theta(n, l)}{\partial l} &= \alpha \left(\left(N_0 + \frac{\theta}{d^2} \right) e^{-d(l+(n-1)T)} (1 - e^{-dT}) - \frac{\theta}{d} T \right) \\ &\doteq \alpha \left(D_{n-1} e^{-dl} - \frac{\theta}{d} T \right), \end{aligned}$$

where $D_{n-1} = (N_0 + \theta/d^2)e^{-d(n-1)T}(1 - e^{-dT})$. Letting $\partial G(n, l)/\partial l = 0$ and solving this equation with respect to l , we get one root, denoted by l^{**} , and

$$l^{**} = \frac{1}{d} \ln \frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} - (n - 1)T.$$

If $D_{n-1} > \theta T/d$, that is,

$$n < 1 + \frac{1}{dT} \ln \left(\frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} \right) \doteq n_\theta,$$

then $l^{**} > 0$, and

$$\begin{aligned} R_{0_{\max}}^N(n, T, l) &= R_0^N(n, T, l^{**}) = R_0(n, T) \exp\{G(n, l^{**})\} \\ &= R_0(n, T) \exp \left\{ -\frac{\alpha}{d} \theta T \left[\frac{T}{2} + \frac{1}{d} \ln \left(\frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} \right) \right] \right\}. \end{aligned}$$

If $D_{n-1} \leq \theta T/d$, then $R_0^N(n, T, l)$ is a decreasing function with respect to $l \in [0, T)$. Thus

$$\begin{aligned} R_{0_{\max}}^N(n, T, l) &= R_0^N(n, T, 0) \\ &= R_0(n, T) \exp \left\{ -\frac{\alpha}{d} \left[\theta T \left(\frac{(2n-1)T}{2} - \frac{1}{d} \right) - \left(N_0 + \frac{\theta}{d^2} \right) e^{-d(n-1)T} (e^{-dT} - 1) \right] \right\}. \end{aligned}$$

By employing the same ideas as for the threshold value $R_{0\delta}^N(n, T, l)$ and the same methods as above, we assume, without loss of generality, that there exists an integer $n' \in \mathcal{N}$ such that (i) $R_0(n, T) \leq R_C$ for $n \leq n'$ due to the high effectiveness of pesticide applications in the initial stage and (ii) $R_0(n, T) > R_C$ for $n > n'$. Thus, we let $\delta_n = \delta_c$ for $n \leq n'$ and let $R_{0\theta_{\max}}^N(n, T, l) = R_C$ for $n > n'$. In the light of the above, we consider the following two cases:

Case 3.3.1. $D_{n-1} > \theta T/d$ (i.e. $n < n_\theta$), i.e. the new number to be released is less than the cumulative death number.

In this case $R_{0\theta_{\max}}^N(n, T, l) = R_{0\theta}^N(n, T, l^{**}) = R_C$. It follows from (12) that we have $G_\theta(n, l^{**}) = \ln(R_C/R_0(n, T))$, i.e.,

$$-\frac{\alpha}{d}\theta T \left[\frac{T}{2} + \frac{1}{d} \ln \left(\frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} \right) \right] = \ln \left(\frac{R_C}{R_0(n, T)} \right). \tag{30}$$

From (30), we have

$$\ln \left(\frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} \right) = -\frac{dT}{2} - \frac{d^2}{\alpha\theta T} \ln \left(\frac{R_C}{R_0(n, T)} \right),$$

i.e.,

$$\frac{d(N_0 + \frac{\theta}{d^2})(1 - e^{-dT})}{\theta T} = e^{-\frac{dT}{2}} \exp \left[-\frac{d^2}{\alpha\theta T} \ln \left(\frac{R_C}{R_0(n, T)} \right) \right] \doteq e^{-\frac{dT}{2}} e^{-\frac{d^2 N_0 A}{\theta}}.$$

Thus,

$$\frac{d(N_0 + \frac{\theta}{d^2})}{\theta T} e^{\frac{d^2 N_0 A}{\theta}} = \frac{e^{-\frac{dT}{2}}}{1 - e^{-dT}}.$$

Further we have

$$\left(\frac{d^2 N_0}{\theta} + 1 \right) e^{\frac{d^2 N_0 A}{\theta}} = \frac{dT e^{-\frac{dT}{2}}}{1 - e^{-dT}}$$

and

$$\left(\frac{Ad^2 N_0}{\theta} + A \right) e^{\frac{d^2 N_0 A}{\theta} + A} = \frac{dT A e^{A - \frac{dT}{2}}}{1 - e^{-dT}}. \tag{31}$$

Solving Eq. (31) with respect to θ , yields two roots

$$\theta_1 = \frac{d^2 N_0 A}{\text{Lambert W}(0, \frac{dT A e^{(A - \frac{dT}{2})}}{1 - e^{-dT}}) - A} \tag{32}$$

and

$$\theta_2 = \frac{d^2 N_0 A}{\text{Lambert W}(-1, \frac{dT A e^{(A - \frac{dT}{2})}}{1 - e^{-dT}}) - A}, \tag{33}$$

where $A = 1/(\alpha T N_0) \ln(R_C/R_0)$, and the Lambert $W(i, x)$ ($i = 0, 1$) function is defined in Appendix B. From this we can prove that only the root $\theta_2 > 0$ is well defined (the proof is presented in Appendix B).

Thus, θ in linear function for all $n \in \mathcal{N}$ can be defined as follows

$$\theta = \begin{cases} \delta_c, & \text{if } n \leq n', \\ \theta_2, & \text{if } n' < n < n_\theta. \end{cases} \tag{34}$$

Thus, the number of natural enemies to be released θt at time t ($t \in [(n - 1)T, nT)$) for Case 3.3.1 can be determined correspondingly.

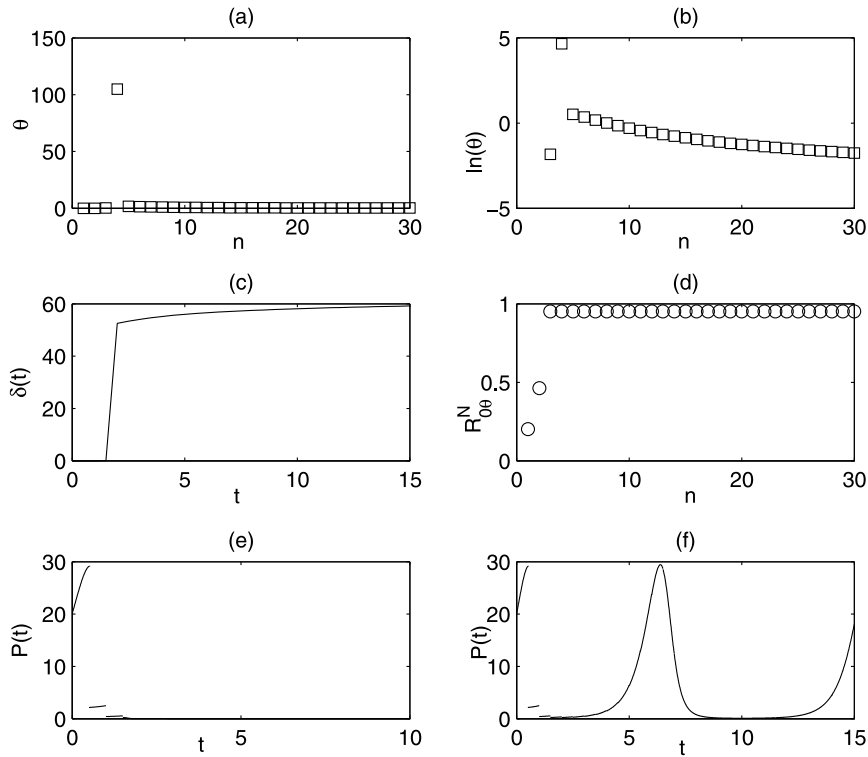


Fig. 2. Calculation of $R_{0\theta}^N(n, T, l)$ and numerical simulations of model (5) with $\delta(t) = \theta t$. The baseline parameter values are as follows: $d_1 = 0.95$; $r = 2$; $d = 0.4$; $\alpha = 0.4$; $\omega_0 = 0.99$, $\beta = 0.2$, $T = 0.5$, $R_C = 0.95$, $\delta_c = 0$, $N_0 = 2$. (a) The plot of θ with respect to n ; (b) The plot of $\ln(\theta)$ with respect to n ; (c) The plot of $\delta(t)$ with respect to t ; (d) The plot of $R_{0\theta}^N(n, T, l)$ with respect to n using the formula (35) to determine the θ ; (e) The time series of the pest population associated with (a); (f) The time series of the pest population without releasing natural enemies.

Case 3.3.2. $D_{n-1} \leq \theta T/d$ (i.e. $n \geq n_\theta$), i.e. the new number of natural enemies to be released is not less than the cumulative death number.

In this case $R_{0\theta_{\max}}^N(n, T, l) = R_{0\theta}^N(n, T, 0) = R_C$. Solving this equation with respect to θ , we get

$$\theta = \frac{-\frac{d}{\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right) - N_0 e^{-d(n-1)T} (1 - e^{-dT})}{T\left(\frac{2n-1}{2}T - \frac{1}{d}\right) + \frac{1-e^{-dT}}{d^2} e^{-d(n-1)T}} \doteq \theta_3.$$

Thus, if $n \geq n_\theta$, then the minimum number of natural enemies released which maintains the threshold value $R_0^N(n, T, l) = R_C$ is determined by $-\frac{2dt}{(2n-1)T^2\alpha} \ln\left(\frac{R_C}{R_0(n, T)}\right)$ at time t .

Therefore, θ in linear function for all $n \in \mathcal{N}$ can be determined as follows:

$$\theta = \begin{cases} \delta_c, & \text{if } n \leq n', \\ \theta_2, & \text{if } n' < n < n_\theta, \\ \theta_3, & \text{if } n \geq n_\theta. \end{cases} \tag{35}$$

Similarly, the number of natural enemies to be released θt at time t ($t \in [(n-1)T, nT)$) for Case 3.3.2 can be determined correspondingly.

To show the effects of the number of natural enemies released which is defined by formula (35) on the pest control with the evolution of pesticide resistance, we carried out numerical investigations. Fig. 2(a, b, c, e) shows that at the initial stage, the chemical strategy alone can successfully control the pest population such that its density decreases very quickly, which indicates that it is not necessary to release natural enemies due

to the high efficacy of the pesticides. Thus, we have $\delta(t) \approx 0$ (i.e. $\theta \approx 0$, as shown in Fig. 2(a, b and c)). As the development of pesticide resistance, in order to maintain the threshold value of $R_{0\theta}^N$ below a constant R_c which is less than one (as shown in Fig. 2(d)), the releasing constant θ and consequently the releasing number of the natural enemies $\delta(t)$ at time t begins to increase very fast in response to pesticide resistance (Fig. 2(a and c)). Otherwise, resurgence must occur after the first application or after several applications of the pesticide, as shown in Fig. 2(f). In the following pesticide application periods, $\delta(t)$ is increasing slowly and eventually tends to a constant and the pest population will die out eventually, as shown in Fig. 2(c) and Fig. 2(e), respectively. At the same time, the threshold value $R_{0\theta}^N$ closes to R_c , a given constant which is less than one.

All these results indicate that only chemical control can restrain pest outbreaks at the initial stage. However, with the development of pesticide resistance, the efficiency of the pesticide becomes weaker and weaker gradually, and a small number of natural enemies cannot restrain the growth of the pest populations after several applications of the pesticide. Therefore, in order to control the pest population below some critical level, farmers should increase the number of natural enemies rapidly in a short time until the number of natural enemies reaches some critical level (at this level, chemical and biological control together can prevent the pest populations from outbreaking again). After that, a small number of natural enemies released could control the pest population successfully, which depends on the accumulation of the number of natural enemies released.

In response to pesticide resistance, the releasing number of natural enemies should vary in accordance with the development of resistance, which is dynamic. Our results shown here can help the design of optimal control tactics aiming to eradicate the pest population or maintain its density below some critical level, for example EIL.

4. Globally attractive pest-free solution and threshold condition

In previous sections, we focused on any pest-free solution defined by (11) and investigated its stability and related biological implications.

However, it follows from the literature [3] that we can find a unique globally attractive solution of subsystem (9). In fact, if we let $u(t) = 1/N(t)$, then the first equation of subsystem (9) can be re-written as

$$\frac{du(t)}{dt} = du(t) \left(1 - \frac{u(t)}{d/\delta(t)} \right), \tag{36}$$

which has a globally attractive solution

$$u(t) = \left(\int_0^\infty e^{-ds} \delta(t-s) ds \right)^{-1}.$$

Consequently, subsystem (9) has a globally attractive solution

$$\bar{N}(t) = \int_0^\infty e^{-ds} \delta(t-s) ds. \tag{37}$$

From which we can see that the pest-free solution of system (5) is given by $(0, \bar{N}(t))$.

Therefore, if we only focus on the pest-free solution $(0, \bar{N}(t))$, then the threshold conditions and analytical formula of δ_n discussed in Section 3 could be much more simple and precise, and we will address these aspects briefly in this section.

Case 4.1.1. Constant step releasing function, i.e. $\delta(t) = \delta_n, t \in [nT, (n + 1)T)$.

Substituting $\delta(t) = \delta_n, t \in [nT, (n + 1)T)$ into (9), we get the globally attractive solution of subsystem (9) for $t \in [nT, (n + 1)T)$

$$\bar{N}_{\delta_n}(t) = \int_0^\infty e^{-ds} \delta(t - s) ds = \frac{\delta_n}{d} (1 - e^{-d(t-nT)}) + \frac{1 - e^{-dT}}{d} \sum_{i=0}^n \delta_{n-i} e^{-d(t-(n-i+1)T)}.$$

It follows from Theorem 2.1 that the threshold value for the global attractivity of the pest free solution $(0, \bar{N}_{\delta_n}(t))$ of system (5) can be obtained, i.e. we have

$$\bar{R}_{0\delta_n}^N(n, T, l) = R_0(n, T) \exp(\bar{G}_{\delta_n}(n, l)),$$

where

$$\bar{G}_{\delta_n}(n, l) = -\frac{\alpha}{d} \left[\left(l - \frac{1 - e^{-dl}}{d} \right) (\delta_n - \delta_{n-1}) + e^{-dl} (1 - e^{-dT}) \left(\bar{N}_{\delta_n}((n - 1)T) - \frac{\delta_{n-1}}{d} \right) + T\delta_{n-1} \right].$$

Further, by using similar methods as those in Case 2.2.1, we can prove that if $\bar{R}_{0\delta_n}^N(n, T, l) \leq 1$, then the pest free solution $(0, \bar{N}_{\delta_n}(t))$ is globally attractive.

Specially, if we choose $\delta(t) = \delta$ for all $t \geq 0$, then

$$\bar{N}(t) = \int_0^\infty e^{-ds} \delta(t - s) ds = \frac{\delta}{d}.$$

Therefore,

$$R_0^N = R_0 \exp \left(-\alpha \int_{l+(n-1)T}^{l+nT} \bar{N}(t) dt \right) = R_0(n, T) e^{-\frac{\delta\alpha T}{d}} \doteq \bar{R}_{0\delta}^N(n, T).$$

Once again, according to Theorem 2.1 we can conclude that the equilibrium $(0, \delta/d)$ of system (5) is globally attractive if $\bar{R}_{0\delta}^N(n, T) \leq 1$.

Letting $\bar{R}_{0\delta}^N(n, T) = R_C$ first and solving this equation with respect to δ , yields

$$\delta = -\frac{d}{\alpha T} \ln \left(\frac{R_C}{R_0(n, T)} \right).$$

Thus, the new number of natural enemies to be released at time t can be defined as

$$\delta = \begin{cases} \delta_c, & \text{if } R_0(n, T) \leq R_C, \\ -\frac{d}{\alpha T} \ln \left(\frac{R_C}{R_0(n, T)} \right), & \text{if } R_0(n, T) > R_C. \end{cases} \tag{38}$$

For the case of $\bar{R}_{0\delta_n}^N(n, T) = R_C$, δ_n can be determined by the similar method as those in Subsection 3.2 due to $\bar{G}_{\delta_n}(n, l)$ and $G_{\delta_n}(n, l)$ having the same formula.

Case 4.1.2. Linear releasing function, i.e. $\delta(t) = \theta t$ for all $t \geq 0$.

Substituting $\delta(t) = \theta t$ into (9), the globally attractive solution of subsystem (9) can be obtained as

$$\bar{N}(t) = \int_0^\infty e^{-ds} \delta(t - s) ds = \frac{\theta}{d} \left(t - \frac{1}{d} \right) \doteq \bar{N}_\theta(t).$$

From a biological point of view, we require $\bar{N}_\theta(t) \geq 0$. Therefore, in the following we only focus on the case $t > 1/d$. Thus,

$$\begin{aligned} G(n, l) &= -\alpha \int_{l+(n-1)T+\frac{1}{d}}^{l+nT+\frac{1}{d}} \bar{N}_\theta(t) dt \\ &= -\frac{\alpha\theta T}{d} \left(\frac{2n-1}{2}T + l \right) \doteq \bar{G}_\theta(n, l), \end{aligned}$$

and $R_0^N(n, T, l) = R_0(n, T) \exp(\bar{G}_\theta(n, l)) \doteq \bar{R}_{0\theta}^N(n, T, l)$. Moreover, the pest free solution $(0, \bar{N}_\theta(t))$ of system (5) is globally attractive if $\bar{R}_{0\theta}^N(n, T, l) \leq 1$ for all $l \in [0, T)$.

It follows from the expression of $\bar{G}_\theta(n, l)$ that this is a decreasing function with respect to l . Therefore,

$$\bar{G}_{\theta_{\max}}(n, l) = \bar{G}_\theta(n, 0) = -\frac{\alpha\theta T^2(2n-1)}{2d},$$

and

$$R_{0_{\max}}^N(n, T, l) = R_0(n, T) \exp(\bar{G}_\theta(n, 0)) = R_0(n, T) \exp\left(-\frac{\alpha\theta T^2(2n-1)}{2d}\right).$$

Letting $R_{0_{\max}}^N(n, T, l) = R_C$ and solving this equation with respect to θ , yields

$$\theta = -\frac{2d}{\alpha T^2(2n-1)} \ln\left(\frac{R_C}{R_0(n, T)}\right).$$

Therefore, the new number of natural enemies to be released at time t can be determined as

$$\delta t = \begin{cases} \delta_c t, & \text{if } n \leq n', \\ -\frac{2dt}{\alpha T^2(2n-1)} \ln\left(\frac{R_C}{R_0(n, T)}\right), & \text{if } n > n', \end{cases} \tag{39}$$

where n' satisfy $R_0(n', T) \leq R_C$ and $R_0(n' + 1, T) > R_C$.

Case 4.1.3. Exponential releasing function, i.e. $\delta(t) = \exp(at)$ for all $t \geq 0$.

In this case, the globally attractive solution of subsystem (9) is given by

$$\bar{N}(t) = \int_0^\infty e^{-ds} \delta(t-s) ds = \frac{1}{a+d} e^{at} \doteq \bar{N}_a(t).$$

By simple calculation we have

$$G(n, l) = -\alpha \int_{l+(n-1)T}^{l+nT} \bar{N}_a(t) dt = -\frac{\alpha e^{a((n-1)T+l)}}{a(a+d)} (e^{aT} - 1).$$

Therefore, the threshold value for global attractivity of the pest free solution $(0, \bar{N}_a(t))$ of system (5) is given by

$$R_0^N(n, T, l) = R_0(n, T, l) \exp\left(\frac{-\alpha}{a(a+d)} e^{a((n-1)T+l)} (e^{aT} - 1)\right) \doteq \bar{R}_{0a}^N(n, T, l)$$

and the pest free solution $(0, \bar{N}_a(t))$ is globally attractive provided that $\bar{R}_{0a}^N(n, T, l) \leq 1$.

It is easy to see that $\bar{R}_{0a}^N(n, T, l)$ is a decreasing function with respect to l , therefore,

$$R_{0\max}^N(n, T, l) = R_0(n, T) \exp\left(\frac{-\alpha}{a(a+d)} e^{a(n-1)T} (e^{aT} - 1)\right).$$

Letting $R_{0\max}^N(n, T, l) = R_C$, we get

$$-\frac{\alpha}{a(a+d)} e^{a(n-1)T} (e^{aT} - 1) = \ln\left(\frac{R_C}{R_0(n, T)}\right).$$

Thus the constant a can be obtained from the above equation, and consequently the new number of natural enemies to be released (i.e. $\exp(at)$) at time t can be determined.

5. Discussion

It is well known that pesticide resistance is a serious issue in pest control. Because chemical control is cheaper, easier to use and more efficient [9,36–38], it is still the main means for pest control. However, frequent sprays of one kind of pesticide may lead to decreasing susceptibility of the pest to the pesticide and then the pest develops resistance to the pesticide, resulting in pest population resurgences and outbreaks.

Once the pest develops resistance to the pesticide, the simple and direct method for successful pest control is switching to another kind of pesticide. However, the pesticide switching method will result in multiple pesticide resistance, and lead to negative effects on pest control. In order to delay the development of pesticide resistance, a reasonable strategy is to adopt the IPM approach [21], that is combining chemical control with other forms of pest control (such as biological control). For example, a pest control strategy is implemented by releasing natural enemies at the same time as pulse spraying of pesticide [17]. In many cases, pesticides act not only on the pest species, but also on the natural enemies, resulting in the resurgence of the pest populations [2,6,26]. In order to reduce the negative impact of pesticide on the natural enemies and maximize the efficiency of biological control, the inoculative release of a small number of natural enemies throughout the pest period is a feasible method. The key question is how to determine the number of natural enemies at every release time, with the aim of letting the density of the pest population getting no larger than some critical level (such as EIL) or eradicating its population, if the strategy of inoculative releases of natural enemies is implemented as pesticide resistance develops.

To address this question, we first developed a pest–natural enemy model in which pulsed actions such as spraying of pesticide, continuous operation with inoculative releasing of natural enemies and the evolution of pesticide resistance were considered. The threshold condition which guarantees the extinction of the pest population for the general strategy of inoculative releasing of natural enemies was given. Specially, the threshold conditions with the aim of eradicating the pest population with three different releasing strategies were derived. For each releasing strategy, the methods and analytical formulae for determining the numbers of natural enemies to be released were investigated and the appropriate analytical expressions derived. Finally, the more detailed threshold conditions for the existence of the globally attractive pest-free solution with three different releasing tactics were discussed. The results obtained in the present work clearly show how to determine the number of natural enemies to be released in response to pesticide resistance, which is dynamic and varies as resistance develops. Therefore, the results shown here can help the design of optimal control tactics in the face of pesticide resistance.

Note that only the simplest pest–natural enemy system has been employed in the present work to address the key issues related to management of pest resistance. However, several factors including functional responses of the natural enemies to the pest, the effects of environmental variations (climate, food availability and so on) on the growth of both pest and natural enemy populations have not been included in the model, which could significantly affect the outcome of pest control [14,5,12,40,8,1,39]. Therefore, more

realistic pest–natural enemy ecosystems with the effects of periodic variation of environments on the growth of pest populations should be considered in the near future aiming to address: how the periodic variation of environments affects the evolution of pest resistance and, consequently, how to determine the number of natural enemies to be released with the pest resistance?

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (GK201402007), by the National Natural Science Foundation of China (NSFC, 11401360, 11371030, 11471201, 11171199), and by General Financial Grant from the China Postdoctoral Science Foundation (2014M552406), the International Development Research Centre (Canada).

Appendix A. The proof of Theorem 2.1

Proof of Theorem 2.1. At the beginning of the proof, we will give the threshold condition about the attraction of the null solution of the following general impulsive differential equation,

$$\begin{cases} \frac{dP(t)}{dt} = P(t)F(t, P(t)), & t \neq nT, \\ P(nT^+) = q(nT)P(nT), & t = nT, \end{cases} \tag{A.1}$$

where $q(nT)$ denotes the survival rate of the pest population after the n -th pesticide application, which depends on $\omega(nT)$, and naturally we have $0 \leq q(nT) < 1$ for all $n \in \mathcal{N}$. Function $F(t, P(t))$ is the growth rate of the pest population including logistic growth as a special case, and we assume that:

- (a) The function F is continuous at $t \neq nT, n \in \mathcal{N}$, and is continuous and differentiable with respect to P , and $\partial F/\partial P$ is continuous with respect to P ;
- (b) There exist two continuous functions φ and λ with $\varphi(P) > 0$ for $P > 0$ and $\lambda(t) \geq 0$ for $t \geq 0$, such that

$$\frac{\partial F}{\partial P}(t, P) \leq -\varphi(P)\lambda(t), \quad \text{for } t \geq 0, P \geq 0,$$

and

$$\int_0^\infty \lambda(t)dt = \infty;$$

- (c) There is a $\beta > 0$ such that
 - (i) $F(t, 0) \leq \beta$ for $t \geq 0$,
 - (ii) $q(nT) \exp(\int_{l+(n-1)T}^{l+nT} F(s, 0)ds) \leq 1$ for $0 \leq l < T$ and $n \in \mathcal{N}$.

Then we have the following lemma.

Lemma 2.2. *Suppose that F in (A.1) satisfies (a), (b) and (c), then for any initial condition $P(0^+) = P_0 > 0$, the solution of (A.1) satisfies $P(t) \rightarrow 0$ as $t \rightarrow \infty$.*

Lemma 2.2 was proved in [17], and it is useful to determine the threshold conditions which guarantee the extinction of the pest population under additional control strategies in our study.

Now we prove [Theorem 2.1](#). It is seen from the second equation of system (5) that $dN(t)/dt > -dN(t) + \delta(t)$. Consider the following impulsive differential equation

$$\begin{cases} \frac{dy(t)}{dt} = -dy(t) + \delta(t), \\ y(0^+) = N_0. \end{cases} \tag{A.2}$$

According to the comparison theorem on differential equations, we can conclude that $N(t) \geq y(t) = N^*(t)$. It follows from the first equation of system (5) that

$$\frac{dP(t)}{dt} \leq rP(t) \left(1 - \frac{P(t)}{K}\right) - \alpha P(t)N^*(t).$$

Now we consider the following impulsive differential equation

$$\begin{cases} \frac{dx(t)}{dt} = rx(t) \left(1 - \frac{x(t)}{K}\right) - \alpha x(t)N^*(t), & t \neq nT, \\ x(nT^+) = (1 - \omega(nT)d_1)x(nT), & t = nT, \\ \frac{d\omega(t)}{dt} = d_1\omega(t)(\omega(t)^{q_n} - 1), \\ x(0^+) = P(0) \doteq P_0. \end{cases} \tag{A.3}$$

Again according to the comparison theorem of impulsive differential equations we have $P(t) \leq x(t)$.

By using the formula of (4) we can easily have

$$\begin{aligned} q(nT) &\doteq 1 - \omega(nT)d_1 \\ &= 1 - \frac{d_1}{(1 + e^{nd_1}((\omega((n-1)T))^{-\frac{n}{T}} - 1))^{\frac{T}{n}}} \end{aligned}$$

and

$$F(s, x) \doteq r - \frac{rx(s)}{K} - \alpha N^*(s).$$

Now we test and verify the conditions of [Lemma 2.2](#). It is easy to see that condition (a) holds true naturally, and

$$\begin{aligned} F(s, 0) &= r - \alpha N^*(s) \leq r, \\ \frac{\partial F(s, x)}{\partial x} &= -\frac{r}{K}, \quad \int_0^\infty \frac{1}{K} ds = \infty. \end{aligned}$$

Therefore,

$$\begin{aligned} \exp\left(\int_{l+(n-1)T}^{l+nT} F(s, 0) ds\right) &= \exp\left(\int_{l+(n-1)T}^{l+nT} r - \alpha N^*(s) ds\right) \\ &= e^{rT} \exp\left(\int_{l+(n-1)T}^{l+nT} (-\alpha N^*(s)) ds\right) \\ &= e^{rT} \exp(G(n, l)). \end{aligned}$$

Thus

$$\begin{aligned} & q(nT) \exp\left(\int_{l+(n-1)T}^{l+nT} F(s, 0) ds\right) \\ &= R_0(n, T) \exp(G(n, l)) \\ &\doteq R_0^N(n, T, l). \end{aligned}$$

According to Lemma 2.2, we can see that if $R_0^N(n, T, l) \leq 1$, then $x(t) \rightarrow 0$ as $t \rightarrow \infty$. Consequently, we have $P(t) \rightarrow 0$ as $t \rightarrow \infty$ provided that $R_0^N(n, T, l) \leq 1$.

Next, we prove that $N(t) \rightarrow N^*(t)$ as $t \rightarrow \infty$. For any $0 < \varepsilon < d/(\alpha\beta)$, there exists a $t_1 > 0$ such that $0 < P(t) < \varepsilon$ for all $t \geq t_1$. Without loss of generality, we may assume that $0 < P(t) < \varepsilon$ holds true for all $t > 0$, then we have

$$-dN(t) + \delta(t) \leq \frac{dN(t)}{dt} \leq (\alpha\beta\varepsilon - d)N(t) + \delta(t).$$

For the left hand inequality, it follows from impulsive differential equation (A.2) that $N(t) \geq y(t) = N^*(t)$. For the right hand inequality, consider the following impulsive differential equation

$$\begin{cases} \frac{dz(t)}{dt} = (\alpha\beta\varepsilon - d)z(t) + \delta(t), \\ z(0^+) = N_0. \end{cases} \quad (\text{A.4})$$

The analytical solution of the above system at any impulsive interval $((n-1)T, nT]$ gives

$$z^*(t) = e^{(\alpha\beta\varepsilon - d)t} \left(N_0 + \int_0^t \delta(s) e^{ds} ds \right). \quad (\text{A.5})$$

Therefore, for any $\varepsilon_1 > 0$, there exists a $t_2 > 0$ such that

$$N^*(t) - \varepsilon_1 < N(t) < z^*(t) + \varepsilon_1$$

for $t > t_2$. Let $\varepsilon \rightarrow 0$, then we have

$$N^*(t) - \varepsilon_1 < N(t) < N^*(t) + \varepsilon_1$$

for $t > t_2$, which indicates that $N(t) \rightarrow N^*(t)$ as $t \rightarrow \infty$. Therefore, the pest-free solution (11) is globally attractive if $R_0^N(n, T, l) \leq 1$. The proof is complete. \square

Appendix B. The definition of Lambert W function and the proof of $\theta_1 < 0$ and $\theta_2 > 0$

The Lambert W function [4] is defined to be the multivalued inverse of the function $z \mapsto z \exp(z)$ satisfying

$$\text{Lambert W}(z) \exp(\text{Lambert W}(z)) = z.$$

First of all, the function $z \exp(z)$ has the positive derivative $(z+1)\exp(z)$ if $z > -1$. Define the inverse function of $z \exp(z)$ restricted on the interval $[-1, \infty)$ to be Lambert W(0, z). Similarly, we define the

inverse function of $z \exp(z)$ restricted on the interval $(-\infty, -1]$ to be Lambert $W(-1, z)$. For more details see Corless et al. [4].

Now we prove that there is only one root θ_2 , which is well defined.

Let

$$f(x) = xe^{-\frac{x}{2}} - 1 + e^{-x},$$

then

$$f(0) = 0 \quad \text{and} \quad f'(x) = \left(1 - \frac{x}{2}\right)e^{-\frac{x}{2}} - e^{-x}.$$

Let

$$g(x) = e^{\frac{x}{2}}f'(x) = 1 - \frac{x}{2} - e^{-\frac{x}{2}},$$

then we have

$$g(0) = 0 \quad \text{and} \quad g'(x) = -\frac{1}{2}(1 - e^{-\frac{x}{2}}).$$

Based on the above, we can see that $g'(x) < 0$ for $x > 0$. This indicates that $g(x)$ is a decreasing function with respect to x for $x > 0$. Therefore, $g(x) < 0$ for $x > 0$. So, $f'(x) < 0$ for $x > 0$, that is $f'(x)$ is a decreasing function with respect to x for $x > 0$. Therefore, $f(x) < 0$ for $x > 0$. This indicates that

$$0 < \frac{dT e^{-\frac{dT}{2}}}{1 - e^{-dT}} < 1.$$

Since $A < 0$, we have

$$\frac{dT A e^{(A - \frac{dT}{2})}}{1 - e^{-dT}} > A e^A > -e^{-1},$$

this indicates that

$$\text{Lambert } W\left(0, \frac{dT A e^{(A - \frac{dT}{2})}}{1 - e^{-dT}}\right) > A$$

and

$$\text{Lambert } W\left(-1, \frac{dT A e^{(A - \frac{dT}{2})}}{1 - e^{-dT}}\right) < A.$$

Therefore, from the expressions of θ_1 and θ_2 , we have $\theta_1 < 0$ and $\theta_2 > 0$. This indicates that only the root θ_2 is well defined, as θ is a nonnegative constant.

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