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THE IMPACT OF MIGRANT WORKERS ON THE TUBERCULOSIS TRANSMISSION: GENERAL MODELS AND A CASE STUDY FOR CHINA

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Abstract. A tuberculosis (TB) transmission model involving migrant workers is proposed and investigated. The basic reproduction number $\mathcal{R}_0$ is calculated, and is shown to be a threshold parameter for the disease to persist or become extinct in the population. The existence and global attractivity of an endemic equilibrium, if $\mathcal{R}_0 > 1$, is also established under some technical conditions. A case study, based on the TB epidemiological and other statistical data in China, indicates that the disease spread can be controlled if effective measures are taken to reduce the reactivation rate of exposed/latent migrant workers. Impact of the migration rate and direction, as well as the duration of home visit stay, on the control of disease spread is also examined numerically.

1. **Introduction.** Tuberculosis (TB) caused by infection with the Mycobacterium tuberculosis (*M. tuberculosis*) is an airborne infectious disease that is preventable and curable [45]. It was estimated that 1.5 million people died from TB in 2006 [45]. In addition, another 200,000 people with HIV died from HIV-associated TB [45]. In the late 1980s, the numbers of incidence of tuberculosis in many Western countries were rising or stagnating after several decades of decrease [15, 21, 23, 36]. The resurgence of tuberculosis might have been attributed to the increase of human mobility, co-infection with HIV, the emergence of drug-resistant strains of *M. tuberculosis*, elimination of TB control programs, and poverty [1, 28, 33].

According to the Ministry of Health of China [30], there are 4.5 million active TB patients, 80% of whom are rural population. There are about 1.5 million new infectious TB cases each year, and about 130,000 deaths are due to TB annually.

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The Ministry of Labor and Social Security of China's web page [31] indicates that there have been about 5 million new migrant workers who leave their poor villages to look for jobs in towns/cities from 1998, while the statistics from the Ministry of Agriculture of China shows that the amount of migrant workers increased to 126 million in 2007, and these workers left their impoverished villages to work in the prosperous towns/cities or south-east coastal cities [2], about 60% of whom flowed into mega cities such as Beijing, Shanghai, Guangzhou and Shenzhen (see the National Bureau of Statistics of China web page [32]). With the heavy influx of migrant workers into cities, curbing the spread of large-scale TB and HIV infection is an immense challenge [16]. As described in [38] on tuberculosis, when over 10% of an entire population is on the move, and when these floating people are poorer and have more tuberculosis than average, public health faces a big problem; and when that happens in China, with a fifth of global population and more than its share of tuberculosis, the world faces a much more difficult public health issue.

Migrant workers in China usually work outside their villages for a long time each year. The total amount of time of migrant workers working in towns/cities ranged from 8 months to 9.4 months during the year 2002 and 2006 (see the web site of the China Labor Market [11, 12]). Most of these migrant workers just return to their homes during the Spring Festival or in the harvest seasons of the year briefly and then go back to work in towns/cities: around 56.6% of migrant workers went back home and reunited with their families during the Spring Festival in the year 2007, and about 83% of whom were planning to return to their former companies in towns/cities to work [12]. This seasonal influx of migrant workers becomes more and more obvious and universal. In addition, there has been an increasing trend that the whole family leaving their villages. In 2006, for example, the amount of migrant workers with their whole families leaving their rural homes accounted for one fifth of the migrant workers [2].

Like many developed countries where immigration is the main reason for stagnation or increasing in TB incidence [3, 7, 17, 49], the resurgence of TB in many parts of China occurs mainly because of the huge population mobility of migrant workers [38]. The migrant population is ranked among the most vulnerable group for TB infection in Chinese metropolitan areas because

(i) 80% of the Chinese TB cases are the rural residents and a substantial portion of migrant workers are infectious or have carried the *M. tuberculosis* before they flow into towns/cities [13, 26];
(ii) comparing with other sub-populations, latent migrant workers are more likely to progress to infectious cases due to heavy working load, malnutrition, and overcrowded living conditions, which affect their immune systems;
(iii) migrant workers are more susceptible to the *M. tuberculosis* infection because of their long frequent contact with infectious migrant workers;
(iv) it is more difficult to identify TB patients and to treat them in a timely fashion due to the lack of periodic health examination for migrant workers;
(v) migrant workers do not have sufficient knowledge on tuberculosis protection and treatment [13, 26, 38].

Many different mathematical models have been developed to consider the impact on TB transmission of factors such as fast and slow progression, drug-resistance, co-infection with HIV, relapse, reinfection, and vaccination [4, 5, 8, 9, 10, 14, 34, 37, 18, 19, 20, 50]. Some mathematical TB models have been formulated to investigate the influence of immigration on the local people [6, 22, 27, 49]. In particular, [49] developed a deterministic discrete-time model of TB transmission in the Canadian born and foreign born populations in order to study the effects of this demographic distinction on the short-term incidence and long-term transmission dynamics, and the impact of immigration latent TB cases on the overall TB incidence rate in the whole community. In [22], a three-population TB model was formulated to examine the impact of latently-infected new immigrations on the TB incidence rate of the host immigration countries and the importance of cross-infection between foreign-born and local-born population in Canada and UK.

Motivated by these studies and the aforementioned situation in China involving a large number of migrant workers, we develop in this paper a TB model with migration to investigate TB transmission in China. The model will incorporate the epidemiological and social and economic features of migrant workers, and our analysis and simulations will allow us to draw both qualitative and quantitative conclusions of how intervention measures corresponding to these features may contribute to a successful national control and prevention program. The rest of the paper is organized as follows. In section 2, we develop the TB model with migration and define the basic reproduction number $\mathcal{R}_0$. In section 3, we study the long-term behavior of the TB model. We prove that there is a unique disease-free equilibrium and the disease always dies out when $\mathcal{R}_0 < 1$; while the disease uniformly persists in the population and there is at least one endemic equilibrium when $\mathcal{R}_0 > 1$. Furthermore, if the migration rates of migrant workers from villages to towns/cities and infectious migrant workers from towns/cities to villages are very small, the global attractivity of the unique endemic equilibrium is also obtained if $\mathcal{R}_0 > 1$. Numerical simulations, provided in section 4, show that the spread of TB may be lowered if the effective actions are taken to reduce the reactivation rate of exposed/latent migrant workers and/or to encourage farmers to stay and work at home. A brief discussion is given in section 5.

2. Model formulation.

In this section, a TB model with migration is developed. The whole population is first divided into three subgroups: rural residents, migrant workers (temporary urban residents), and urban population (permanent urban residents). Each subgroup is further subdivided into four compartments: susceptible (S), exposed/latent infection (E), infectious (I), and recovery/treated (R). The subscripts $r, m,$ and u denote rural residents, migrant workers, and urban population, respectively.

The susceptible individuals can be infected by the frequent contact with infectious persons and enter the infectious classes by fast developing TB cases, or just flow into the exposed/latent classes containing the bacteria, who when their immune systems are weakened will easily reactivate to infectious TB cases. The directly observed treatment, short-course (DOTS) strategy can be used to treat the infectious TB cases and the completely cured TB patients will go into the recovery/treated classes.

All the infectious TB cases can infect the susceptible individuals in the same subgroup. Migrant workers travel between villages and towns/cities. When the harvest season or the Spring Festival comes, these migrant workers go home, reunite with their families, and become the members of rural residents. When the infectious migrant workers (I_m) go home, they immediately become the infectious rural residents (I_r). Thus, we just consider the infectivity between the susceptible rural residents (S_r) and the infectious rural residents (I_r) after those infectious migrant workers (I_m) get back to their homes. Because the migrant workers live in the same regions with the urban population after they return to towns/cities in order to find job, they have contact with urban population. Therefore, the infectious migrant workers (I_m) can infect the susceptible urban population (S_u), and vice versa, the infectious urban population (I_u) can also infect susceptible migrant workers (S_m) (see Fig. 1).

We assume that all the newborns are left in their villages with their grandparents or other relatives. Furthermore, there is no death or birth during travel. The mass action incidence is used here. Thus, our TB model involving migrant workers is described by the following ordinary differential system:

$$
\begin{aligned}
\frac{dS_r}{dt} &= \Lambda_r - \beta_{rr}S_rI_r - (\mu + a_{mr})S_r + a_{rm}S_m,\\
\frac{dE_r}{dt} &= (1-p_r)\beta_{rr}S_rI_r - (\mu + k_r + b_{mr})E_r + b_{rm}E_m,\\
\frac{dI_r}{dt} &= p_r\beta_{rr}S_rI_r + k_rE_r - (\mu + \alpha_r + \gamma_r + c_{mr})I_r + c_{rm}I_m,\\
\frac{dR_r}{dt} &= \gamma_rI_r - (\mu + e_{mr})R_r + e_{rm}R_m,\\
\frac{dS_m}{dt} &= -\beta_{mm}S_mI_m - \beta_{mu}S_mI_u - (\mu + a_{rm})S_m + a_{mr}S_r,\\
\frac{dE_m}{dt} &= (1-p_m)S_m(\beta_{mm}I_m + \beta_{mu}I_u) - (\mu + k_m + b_{rm})E_m + b_{mr}E_r,\\
\frac{dI_m}{dt} &= p_mS_m(\beta_{mm}I_m + \beta_{mu}I_u) + k_mE_m - (\mu + \alpha_m + \gamma_m + c_{rm})I_m + c_{mr}I_r,\\
\frac{dR_m}{dt} &= \gamma_mI_m - (\mu + e_{rm})R_m + e_{mr}R_r, \qquad (1)\\
\frac{dS_u}{dt} &= \Lambda_u - \beta_{uu}S_uI_u - \beta_{um}S_uI_m - \mu S_u,\\
\frac{dE_u}{dt} &= (1-p_u)S_u(\beta_{uu}I_u + \beta_{um}I_m) - (\mu + k_u)E_u,\\
\frac{dI_u}{dt} &= p_uS_u(\beta_{uu}I_u + \beta_{um}I_m) + k_uE_u - (\mu + \alpha_u + \gamma_u)I_u,\\
\frac{dR_u}{dt} &= \gamma_uI_u - \mu R_u,\\
N_i &= S_i + E_i + I_i + R_i,\ i\in\{r,m,u\},\ \text{and } N = N_r + N_m + N_u.
\end{aligned}
$$

Note that $N_i(t)$, $i\in\{r,m,u\}$, represents the number of the subpopulation in the ith subgroup at time t. $N(t)$ is the number of the whole population at time t.

Parameters used in the model are defined as follows. $\Lambda_i, i\in\{r,u\}$, represents the recruitment rate of the population in the ith subgroup. β_{rr} is the transmission rate between susceptible rural residents and infectious rural residents. $\beta_{ij}, i,j\in\{m,u\}$, represents the transmission coefficient from the infectious individuals in the jth subgroup to the susceptible individuals in the ith subgroup. μ is the natural death rate of the whole population. $a_{ij}, i,j\in\{r,m\}, i\neq j$, represents the migration rate of susceptible individuals from the jth subgroup to the ith subgroup. $b_{ij}, i,j\in\{r,m\}, i\neq j$, represents the migration rate of exposed/latent individuals from the jth subgroup to the ith subgroup. $c_{ij}, i,j\in\{r,m\}, i\neq j$, represents the migration rate of the infectious individuals from the jth subgroup to the ith subgroup. $e_{ij}, i,j\in\{r,m\}, i\neq j$, represents the migration rate of recovery/treated individuals from the jth subgroup to the ith subgroup. $p_i, i\in\{r,m,u\}$, is the fraction of the newly infected individuals who progress to infectious TB cases within the first two years after infection in the ith subgroup. $k_i, i\in\{r,m,u\}$, represents the reactivation rate to the infectious TB cases in the ith subgroup. $\alpha_i, i\in\{r,m,u\}$, is the disease-induced death rate of the infectious individuals in the ith subgroup. $\gamma_i, i\in\{r,m,u\}$, is the removal/treatment rate in the ith subgroup. All these parameters are positive and $0 < p_i < 1, i\in\{r,m,u\}$.

Clearly, the right hand side of system (1) is continuously differentiable on the domain R_+^{12}. By [39, Theorem 5.2.1], it follows that for any initial value in R_+^{12}, there is a unique nonnegative solution on its maximal interval of existence. Adding

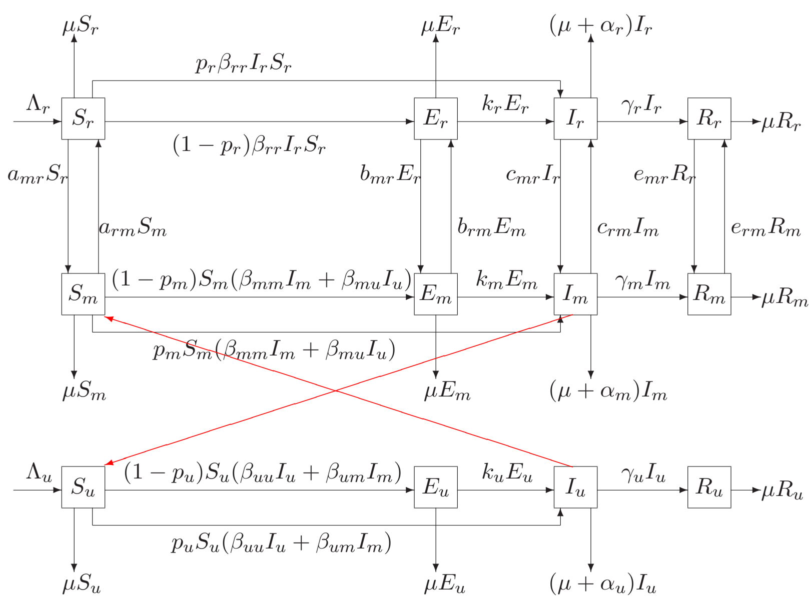

FIGURE 1. The schematic diagram of TB transmission involving migrant workers.

the first twelve equations of system (1) gives

$$\frac{dN}{dt} = \Lambda_r + \Lambda_u - \mu N - \alpha_r I_r - \alpha_m I_m - \alpha_u I_u \le \Lambda_r + \Lambda_u - \mu N. \tag{2}$$

Then $N(t)\le N(0)e^{-\mu t}+(\Lambda_r+\Lambda_u)(1-e^{-\mu t})/\mu \le N(0)+(\Lambda_r+\Lambda_u)/\mu, \forall\, t\ge 0$. Therefore, all the solutions of system (1) exist globally on the interval $[0,+\infty)$. Since the equations for $R_r, R_m,$ and R_u are decoupled from other equations of system (1), it suffices to study the following subsystem:

$$\begin{aligned}
\tfrac{dS_r}{dt} &= \Lambda_r - \beta_{rr}S_rI_r - (\mu+a_{mr})S_r + a_{rm}S_m,\\
\tfrac{dS_m}{dt} &= -\beta_{mm}S_mI_m - \beta_{mu}S_mI_u - (\mu+a_{rm})S_m + a_{mr}S_r,\\
\tfrac{dS_u}{dt} &= \Lambda_u - \beta_{uu}S_uI_u - \beta_{um}S_uI_m - \mu S_u,\\
\tfrac{dE_r}{dt} &= (1-p_r)\beta_{rr}S_rI_r - (\mu+k_r+b_{mr})E_r + b_{rm}E_m,\\
\tfrac{dE_m}{dt} &= (1-p_m)S_m(\beta_{mm}I_m+\beta_{mu}I_u) - (\mu+k_m+b_{rm})E_m + b_{mr}E_r,\\
\tfrac{dE_u}{dt} &= (1-p_u)S_u(\beta_{uu}I_u+\beta_{um}I_m) - (\mu+k_u)E_u,\\
\tfrac{dI_r}{dt} &= p_r\beta_{rr}S_rI_r + k_rE_r - (\mu+\alpha_r+\gamma_r+c_{mr})I_r + c_{rm}I_m,\\
\tfrac{dI_m}{dt} &= p_mS_m(\beta_{mm}I_m+\beta_{mu}I_u) + k_mE_m - (\mu+\alpha_m+\gamma_m+c_{rm})I_m + c_{mr}I_r,\\
\tfrac{dI_u}{dt} &= p_uS_u(\beta_{uu}I_u+\beta_{um}I_m) + k_uE_u - (\mu+\alpha_u+\gamma_u)I_u.
\end{aligned} \tag{3}$$

In order to determine the basic reproduction number $\mathcal{R}_0$ of system (3), we first consider the following system:

$$\begin{aligned} \frac{dS_r}{dt} &= \Lambda_r - (\mu + a_{mr})S_r + a_{rm}S_m, \\ \frac{dS_m}{dt} &= -(\mu + a_{rm})S_m + a_{mr}S_r, \\ \frac{dS_u}{dt} &= \Lambda_u - \mu S_u. \end{aligned} \tag{4}$$

It is easy to see that system (4) has a unique positive equilibrium $\tilde{S}^* = (S_r^*, S_m^*, S_u^*)$, where

$$S_r^* = \frac{\Lambda_r(\mu + a_{rm})}{\mu^2 + \mu(a_{rm} + a_{mr})}, \quad S_m^* = \frac{\Lambda_r a_{mr}}{\mu^2 + \mu(a_{rm} + a_{mr})}, \quad S_u^* = \frac{\Lambda_u}{\mu},$$

and $\tilde{S}^*$ is globally asymptotically stable for system (4) in R_+^3. Thus, system (3) has a unique disease-free equilibrium $P_0 = (S_r^*, S_m^*, S_u^*, 0, 0, 0, 0, 0, 0)$.

According to the definitions of the next generation matrix and the basic reproduction number [42], we define

$$F_1 = \begin{bmatrix} 0 & 0 & 0 & (1-p_r)\beta_{rr}S_r^* & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-p_m)\beta_{mm}S_m^* & (1-p_m)\beta_{mu}S_m^* \\ 0 & 0 & 0 & 0 & (1-p_u)\beta_{um}S_u^* & (1-p_u)\beta_{uu}S_u^* \\ 0 & 0 & 0 & p_r\beta_{rr}S_r^* & 0 & 0 \\ 0 & 0 & 0 & 0 & p_m\beta_{mm}S_m^* & p_m\beta_{mu}S_m^* \\ 0 & 0 & 0 & 0 & p_u\beta_{um}S_u^* & p_u\beta_{uu}S_u^* \end{bmatrix},$$

and

$$V_1 = \begin{bmatrix} l_1 & -b_{rm} & 0 & 0 & 0 & 0 \\ -b_{mr} & l_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu + k_u & 0 & 0 & 0 \\ -k_r & 0 & 0 & l_3 & -c_{rm} & 0 \\ 0 & -k_m & 0 & -c_{mr} & l_4 & 0 \\ 0 & 0 & -k_u & 0 & 0 & l_5 \end{bmatrix},$$

where $l_1 = \mu + k_r + b_{mr}, l_2 = \mu + k_m + b_{rm}, l_3 = \mu + \alpha_r + \gamma_r + c_{mr}, l_4 = \mu + \alpha_m + \gamma_m + c_{rm}$, and $l_5 = \mu + \alpha_u + \gamma_u$.

Therefore, the basic reproduction number is defined as $\mathcal{R}_0 = \rho(F_1V_1^{-1})$, where $\rho(M)$ denotes the spectral radius of matrix M. The proof of [42, Theorem 2] implies the following result.

Lemma 2.1. *Let $M_1 = F_1 - V_1$ and $s(M_1)$ be the maximum real part of all the eigenvalues of the matrix M_1. Then $s(M_1) < 0$ if and only if $\mathcal{R}_0 < 1$, and $s(M_1) > 0$ if and only if $\mathcal{R}_0 > 1$.*

3. **The threshold dynamics.** In this section, we show that the disease-free equilibrium P_0 is globally asymptotically attractive when $\mathcal{R}_0 < 1$ and the disease is uniformly persistent when $\mathcal{R}_0 > 1$. Furthermore, we show that when $\mathcal{R}_0 > 1$, the system has a unique globally attractive endemic equilibrium provided the migration rates of migrant workers from villages to towns/cities and of infectious migrant workers from towns/cities to villages are very small.

For convenience, the solution $(S_r(t), S_m(t), S_u(t), E_r(t), E_m(t), E_u(t), I_r(t), I_m(t), I_u(t))$ of system (3) is denoted by $(S(t), E(t), I(t))$. Let $\Phi_t : R_+^9 \to R_+^9$ be the solution semiflow of system (3), that is, $\Phi_t(S(0), E(0), I(0)) = (S(t), E(t), I(t))$ is the solution of system (3) with the initial value $(S(0), E(0), I(0))$. It is easy to see that the compact set

$$\begin{aligned}\Omega: \ = \ & \{(S_r, S_m, S_u, E_r, E_m, E_u, I_r, I_m, I_u) \in R^9_+ \ : \ S_r + E_r + I_r \\ & + S_m + E_m + I_m \leq \frac{\Lambda_r}{\mu}, \text{ and } S_u + E_u + I_u \leq \frac{\Lambda_u}{\mu}\}\end{aligned}$$

is positively invariant for Φ_t, and attracts all forward orbits of Φ_t in R^9_+.

Theorem 3.1. *If $\mathcal{R}_0 < 1$, P_0 is globally asymptotically stable; while if $\mathcal{R}_0 > 1$, P_0 is unstable and there exists a positive constant ζ such that every solution $(S(t), E(t), I(t))$ of system (3) with initial value $(S(0), E(0), I(0)) \in R^3_+ \times Int\ (R^6_+)$ satisfies*

$$\liminf_{t\to\infty} E_i(t) \geq \zeta, \ \liminf_{t\to\infty} I_i(t) \geq \zeta, \ i \in \{r, m, u\},$$

and system (3) admits at least one endemic equilibrium.

Proof. We first consider the case of $\mathcal{R}_0 < 1$. By [42, Theorem 2], P_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$. Thus, it is sufficient to prove the global attractivity of P_0 when $\mathcal{R}_0 < 1$. In view of system (3), we have

$$\begin{aligned}\tfrac{dS_r}{dt} & \leq \Lambda_r - (\mu + a_{mr})S_r + a_{rm}S_m, \\ \tfrac{dS_m}{dt} & \leq -(\mu + a_{rm})S_m + a_{mr}S_r, \\ \tfrac{dS_u}{dt} & \leq \Lambda_u - \mu S_u.\end{aligned} \tag{5}$$

By the aforementioned conclusion for system (4) and the comparison principle of cooperative systems [40, Theorem B.1], it follows that for any $\varepsilon > 0$, we have $S_i(t) < S^*_i + \varepsilon$, $i \in \{r, m, u\}$, for sufficiently large t. Thus, if t is sufficiently large, we get

$$\begin{aligned}\tfrac{dE_r}{dt} & < (1 - p_r)\beta_{rr}(S^*_r + \varepsilon)I_r - (\mu + k_r + b_{mr})E_r + b_{rm}E_m, \\ \tfrac{dE_m}{dt} & < (1 - p_m)(S^*_m + \varepsilon)(\beta_{mm}I_m + \beta_{mu}I_u) - (\mu + k_m + b_{rm})E_m + b_{mr}E_r, \\ \tfrac{dE_u}{dt} & < (1 - p_u)(S^*_u + \varepsilon)(\beta_{uu}I_u + \beta_{um}I_m) - (\mu + k_u)E_u, \\ \tfrac{dI_r}{dt} & < p_r\beta_{rr}(S^*_r + \varepsilon)I_r + k_rE_r - (\mu + \alpha_r + \gamma_r + c_{mr})I_r + c_{rm}I_m, \\ \tfrac{dI_m}{dt} & < p_m(S^*_m + \varepsilon)(\beta_{mm}I_m + \beta_{mu}I_u) + k_mE_m \\ & \quad -(\mu + \alpha_m + \gamma_m + c_{rm})I_m + c_{mr}I_r, \\ \tfrac{dI_u}{dt} & < p_u(S^*_u + \varepsilon)(\beta_{uu}I_u + \beta_{um}I_m) + k_uE_u - (\mu + \alpha_u + \gamma_u)I_u.\end{aligned} \tag{6}$$

Thus, it suffices to prove that the solutions of the following auxiliary system

$$\begin{aligned}\tfrac{d\hat{E}_r}{dt} & = (1 - p_r)\beta_{rr}(S^*_r + \varepsilon)\hat{I}_r - (\mu + k_r + b_{mr})\hat{E}_r + b_{rm}\hat{E}_m, \\ \tfrac{d\hat{E}_m}{dt} & = (1 - p_m)(S^*_m + \varepsilon)(\beta_{mm}\hat{I}_m + \beta_{mu}\hat{I}_u) - (\mu + k_m + b_{rm})\hat{E}_m + b_{mr}\hat{E}_r, \\ \tfrac{d\hat{E}_u}{dt} & = (1 - p_u)(S^*_u + \varepsilon)(\beta_{uu}\hat{I}_u + \beta_{um}\hat{I}_m) - (\mu + k_u)\hat{E}_u, \\ \tfrac{d\hat{I}_r}{dt} & = p_r\beta_{rr}(S^*_r + \varepsilon)\hat{I}_r + k_r\hat{E}_r - (\mu + \alpha_r + \gamma_r + c_{mr})\hat{I}_r + c_{rm}\hat{I}_m, \\ \tfrac{d\hat{I}_m}{dt} & = p_m(S^*_m + \varepsilon)(\beta_{mm}\hat{I}_m + \beta_{mu}\hat{I}_u) + k_m\hat{E}_m \\ & \quad -(\mu + \alpha_m + \gamma_m + c_{rm})\hat{I}_m + c_{mr}\hat{I}_r, \\ \tfrac{d\hat{I}_u}{dt} & = p_u(S^*_u + \varepsilon)(\beta_{uu}\hat{I}_u + \beta_{um}\hat{I}_m) + k_u\hat{E}_u - (\mu + \alpha_u + \gamma_u)\hat{I}_u,\end{aligned} \tag{7}$$

tend to zero as t approaches to infinity. Let M_2 be the matrix defined by

$$M_2 = \begin{bmatrix} 0 & 0 & 0 & (1-p_r)\beta_{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-p_m)\beta_{mm} & (1-p_m)\beta_{mu} \\ 0 & 0 & 0 & 0 & (1-p_u)\beta_{um} & (1-p_u)\beta_{uu} \\ 0 & 0 & 0 & p_r\beta_{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_m\beta_{mm} & p_m\beta_{mu} \\ 0 & 0 & 0 & 0 & p_u\beta_{um} & p_u\beta_{uu} \end{bmatrix}.$$

Since $\mathcal{R}_0 < 1$, Lemma 2.1 implies $s(M_1) < 0$. By the continuity of $s(M_1+\varepsilon M_2)$ in ε, we can choose ε small enough so that $s(M_1+\varepsilon M_2) < 0$. Consequently, the solutions of system (7) tend to zero as t goes to infinity. By the comparison principle of cooperative systems [40, Theorem B.1], we have $(E_r(t), E_m(t), E_u(t), I_r(t), I_m(t), I_u(t)) \to 0$ as $t\to\infty$. By the theory of asymptotically autonomous systems [41, Theorem 1.2], it then follows that $\lim_{t\to\infty} S_i(t) = S_i^*, i\in\{r, m, u\}$.

In the case where $\mathcal{R}_0 > 1$, it follows from [42, Theorem 2] that P_0 is unstable. Define

$$\begin{aligned} X &= \{(S_r, S_m, S_u, E_r, E_m, E_u, I_r, I_m, I_u) \ : \ S_i\geq 0, E_i\geq 0, I_i\geq 0, i = r, m, u\}, \\ X_0 &= \{(S_r, S_m, S_u, E_r, E_m, E_u, I_r, I_m, I_u)\in X \ : \ E_i > 0, I_i > 0, i = r, m, u\}, \\ \text{and} \quad \partial X_0 &= X\setminus X_0. \end{aligned}$$

We first prove system (3) is uniformly persistent with respect to X_0. Note that X and X_0 are positively invariant sets, X_0 is relatively open in X, and ∂X_0 is relatively closed in X. Since solutions of system (3) are ultimately bounded and uniformly bounded, Φ_t has a global compact attractor in X. Set

$$M_\partial := \{(S(0), E(0), I(0))\in\partial X_0 : \ \Phi_t(S(0), E(0), I(0))\in\partial X_0, \forall t\geq 0\}.$$

We now show that

$$M_\partial = \{(S, 0, 0) : \ S\geq 0\}. \tag{8}$$

Clearly, $\{(S, 0, 0) : \ S\geq 0\}\subset M_\partial$. Assume that $(S(0), E(0), I(0))\in M_\partial$. It suffices to show that $(E(t), I(t)) = 0, \forall t\geq 0$. Suppose it is not true. Then there exists a $\hat{t}_1\geq 0$ such that $(E(\hat{t}_1), I(\hat{t}_1)) > 0$, that is, $(E(\hat{t}_1), I(\hat{t}_1)) \in R_+^2 \setminus \{0\}$. Next, we prove that for the initial value $(S(\hat{t}_1), E(\hat{t}_1), I(\hat{t}_1))\in X$, there is an $m^* > 0$ such that $S(t)\geq m^*, \forall t\in[\hat{t}_1, \hat{t}_1 + 1]$. Integrating both sides of the third equation of (3) gives

$$\begin{aligned} S_u(t) &= e^{-\int_{\hat{t}_1}^t a(s_1)ds_1} \left(S_u(\hat{t}_1) + \Lambda_u \int_{\hat{t}_1}^t e^{\int_{\hat{t}_1}^{s_2} a(s_1)ds_1} ds_2 \right) \\ &\geq \Lambda_u e^{-\int_{\hat{t}_1}^t a(s_1)ds_1} \int_{\hat{t}_1}^t e^{\int_{\hat{t}_1}^{s_2} a(s_1)ds_1} ds_2, \quad \forall t\in[\hat{t}_1, \hat{t}_1 + 1], \end{aligned}$$

where $a(t) = \mu + \beta_{uu}I_u(t) + \beta_{um}I_m(t)$. Thus, there exists an $m_1 > 0$ so that $S_u(t)\geq m_1, \forall t\in[\hat{t}_1, \hat{t}_1 + 1]$. By the first equation of (3), we get

$$\frac{dS_r(t)}{dt}\geq\Lambda_r - (\mu + a_{mr} + \beta_{rr}I_r(t))S_r(t), \quad \forall t\in[\hat{t}_1, \hat{t}_1 + 1].$$

By a similar procedure, we know that there is an $m_2 > 0$ so that $S_r(t)\geq m_2$, $\forall t\in[\hat{t}_1, \hat{t}_1 + 1]$. The second equation of system (3) gives

$$\frac{dS_m(t)}{dt}\geq a_{mr}m_2 - (\mu + a_{rm} + \beta_{mm}I_m(t) + \beta_{mu}I_u(t))S_m(t), \quad \forall t\in[\hat{t}_1, \hat{t}_1 + 1].$$

A similar procedure implies that there exists an $m_3 > 0$ such that $S_m(t) \geq m_3$, $\forall t \in [\hat{t}_1, \hat{t}_1+1]$. Denote $m^* := \min\{m_1, m_2, m_3\}$, then $m^* > 0$ and $S(t) \geq m^*$, $\forall t \in [\hat{t}_1, \hat{t}_1+1]$. For any $t \in [\hat{t}_1, \hat{t}_1+1]$, system (3) yields

$$\begin{aligned}
\frac{dE_r}{dt} &\geq (1-p_r)\beta_{rr} m^* I_r - (\mu + k_r + b_{mr})E_r + b_{rm}E_m,\\
\frac{dE_m}{dt} &\geq (1-p_m) m^* (\beta_{mm} I_m + \beta_{mu} I_u) - (\mu + k_m + b_{rm})E_m + b_{mr}E_r,\\
\frac{dE_u}{dt} &\geq (1-p_u) m^* (\beta_{uu} I_u + \beta_{um} I_m) - (\mu + k_u)E_u,\\
\frac{dI_r}{dt} &\geq p_r \beta_{rr} m^* I_r + k_r E_r - (\mu + \alpha_r + \gamma_r + c_{mr})I_r + c_{rm} I_m,\\
\frac{dI_m}{dt} &\geq p_m m^* (\beta_{mm} I_m + \beta_{mu} I_u) + k_m E_m - (\mu + \alpha_m + \gamma_m + c_{rm})I_m + c_{mr} I_r,\\
\frac{dI_u}{dt} &\geq p_u m^* (\beta_{uu} I_u + \beta_{um} I_m) + k_u E_u - (\mu + \alpha_u + \gamma_u) I_u.
\end{aligned} \tag{9}$$

For any $t \in [\hat{t}_1, \hat{t}_1+1]$, consider the following auxiliary system

$$\begin{aligned}
\frac{d\hat{E}_r}{dt} &= (1-p_r)\beta_{rr} m^* \hat{I}_r - (\mu + k_r + b_{mr})\hat{E}_r + b_{rm}\hat{E}_m,\\
\frac{d\hat{E}_m}{dt} &= (1-p_m) m^* (\beta_{mm} \hat{I}_m + \beta_{mu} \hat{I}_u) - (\mu + k_m + b_{rm})\hat{E}_m + b_{mr}\hat{E}_r,\\
\frac{d\hat{E}_u}{dt} &= (1-p_u) m^* (\beta_{uu} \hat{I}_u + \beta_{um} \hat{I}_m) - (\mu + k_u)\hat{E}_u,\\
\frac{d\hat{I}_r}{dt} &= p_r \beta_{rr} m^* \hat{I}_r + k_r \hat{E}_r - (\mu + \alpha_r + \gamma_r + c_{mr})\hat{I}_r + c_{rm} \hat{I}_m,\\
\frac{d\hat{I}_m}{dt} &= p_m m^* (\beta_{mm} \hat{I}_m + \beta_{mu} \hat{I}_u) + k_m \hat{E}_m - (\mu + \alpha_m + \gamma_m + c_{rm})\hat{I}_m + c_{mr} \hat{I}_r,\\
\frac{d\hat{I}_u}{dt} &= p_u m^* (\beta_{uu} \hat{I}_u + \beta_{um} \hat{I}_m) + k_u \hat{E}_u - (\mu + \alpha_u + \gamma_u) \hat{I}_u,
\end{aligned} \tag{10}$$

with the initial value $(E(\hat{t}_1), I(\hat{t}_1)) > 0$. Note that the Jacobian matrix of the right hand side of system (10) is cooperative and irreducible. Then [39, Theorem 4.1.1] implies that $(\hat{E}(t), \hat{I}(t)) \gg 0$, $\forall t \in (\hat{t}_1, \hat{t}_1+1]$. By [40, Theorem B.1], it follows that the solution of system (9) with the initial value $(E(\hat{t}_1), I(\hat{t}_1)) > 0$ satisfies $(E(t), I(t)) \gg 0$, $\forall t \in (\hat{t}_1, \hat{t}_1+1]$. Thus, we have $(S(t), E(t), I(t)) \in X_0$, $\forall t \in (\hat{t}_1, \hat{t}_1+1]$, and hence, $(S(t), E(t), I(t)) \in X_0$, $\forall t > \hat{t}_1$, which contradicts the assumption that $(S(t), E(t), I(t)) \in \partial X_0$, $\forall t \geq 0$. This proves (8). Clearly, there is exactly one equilibrium P_0 which is globally asymptotically attractive in M_∂.

Since $\mathcal{R}_0 > 1$ implies $s(M_1) > 0$, we can choose $\eta > 0$ small enough so that $s(M_1 - \eta M_2) > 0$. We consider the following perturbed system associated with system (4):

$$\begin{aligned}
\frac{dS_r}{dt} &= \Lambda_r - (\mu + a_{mr} + \beta_{rr}\varepsilon_1)S_r + a_{rm}S_m,\\
\frac{dS_m}{dt} &= -(\mu + a_{rm} + \beta_{mm}\varepsilon_1 + \beta_{mu}\varepsilon_1)S_m + a_{mr}S_r,\\
\frac{dS_u}{dt} &= \Lambda_u - (\mu + \beta_{uu}\varepsilon_1 + \beta_{um}\varepsilon_1)S_u.
\end{aligned} \tag{11}$$

Note that system (11) admits a unique globally asymptotically stable equilibrium $\tilde{S}^*(\varepsilon_1) = (S_r^*(\varepsilon_1), S_m^*(\varepsilon_1), S_u^*(\varepsilon_1))$, where

$$S_r^*(\varepsilon_1) = \frac{\Lambda_r(\mu + a_{rm} + \varepsilon_1(\beta_{mm} + \beta_{mu}))}{\mu^2 + \mu(a_{rm} + a_{mr}) + \varepsilon_1 B},\ S_m^*(\varepsilon_1) = \frac{\Lambda_r a_{mr}}{\mu^2 + \mu(a_{rm} + a_{mr}) + \varepsilon_1 B},$$

$$B := (\mu + a_{mr})(\beta_{mm} + \beta_{mu} + \beta_{rr}) + \varepsilon_1 \beta_{rr}(\beta_{mm} + \beta_{mu}),\ S_u^*(\varepsilon_1) = \frac{\Lambda_u}{\mu + \varepsilon_1(\beta_{uu} + \beta_{um})},$$

and $\tilde{S}^*(\varepsilon_1)$ is continuous in ε_1 and $\tilde{S}^*(\varepsilon_1) \to \tilde{S}^*$ as $\varepsilon_1 \to 0^+$. Thus, we can further restrict ε_1 small enough so that $\tilde{S}^*(\varepsilon_1) > \tilde{S}^* - (\eta, \eta, \eta)$. For any solution

$(S(t), E(t), I(t))$ with initial value $(S(0), E(0), I(0)) \in X_0$ of system (3), we further claim that

$$\limsup_{t\to\infty} \max_{i\in\{r,m,u\}} \{E_i(t), I_i(t)\} > \varepsilon_1. \tag{12}$$

Suppose that there exists a $T > 0$ such that $E_i(t) \leq \varepsilon_1, I_i(t) \leq \varepsilon_1, i \in \{r, m, u\}$, for all $t \geq T$. Then for all $t \geq T$, we have

$$\begin{aligned} \frac{dS_r}{dt} &\geq \Lambda_r - (\mu + a_{mr} + \beta_{rr}\varepsilon_1)S_r + a_{rm}S_m, \\ \frac{dS_m}{dt} &\geq -(\mu + a_{rm} + \beta_{mm}\varepsilon_1 + \beta_{mu}\varepsilon_1)S_m + a_{mr}S_r, \\ \frac{dS_u}{dt} &\geq \Lambda_u - (\mu + \beta_{uu}\varepsilon_1 + \beta_{um}\varepsilon_1)S_u. \end{aligned} \tag{13}$$

Since the equilibrium $\tilde{S}^*(\varepsilon_1)$ of system (11) is globally asymptotically stable and $\tilde{S}^*(\varepsilon_1) > \tilde{S}^* - (\eta, \eta, \eta)$, there is a $T_1 > T$ such that the solution $S(t)$ of system (13) satisfies $S(t) \geq \tilde{S}^* - (\eta, \eta, \eta)$ for all $t \geq T_1$. Indeed, for all $t \geq T_1$, there holds

$$\begin{aligned} \frac{dE_r}{dt} &\geq (1 - p_r)\beta_{rr}(S_r^* - \eta)I_r - (\mu + k_r + b_{mr})E_r + b_{rm}E_m, \\ \frac{dE_m}{dt} &\geq (1 - p_m)(S_m^* - \eta)(\beta_{mm}I_m + \beta_{mu}I_u) - (\mu + k_m + b_{rm})E_m + b_{mr}E_r, \\ \frac{dE_u}{dt} &\geq (1 - p_u)(S_u^* - \eta)(\beta_{uu}I_u + \beta_{um}I_m) - (\mu + k_u)E_u, \\ \frac{dI_r}{dt} &\geq p_r\beta_{rr}(S_r^* - \eta)I_r + k_rE_r - (\mu + \alpha_r + \gamma_r + c_{mr})I_r + c_{rm}I_m, \\ \frac{dI_m}{dt} &\geq p_m(S_m^* - \eta)(\beta_{mm}I_m + \beta_{mu}I_u) + k_mE_m \\ &\quad -(\mu + \alpha_m + \gamma_m + c_{rm})I_m + c_{mr}I_r, \\ \frac{dI_u}{dt} &\geq p_u(S_u^* - \eta)(\beta_{uu}I_u + \beta_{um}I_m) + k_uE_u - (\mu + \alpha_u + \gamma_u)I_u. \end{aligned} \tag{14}$$

Note that the matrix $M_1 - \eta M_2$ is irreducible and has nonnegative off-diagonal elements. By [40, Theorem A.5], $s(M_1 - \eta M_2)$ is a positive simple eigenvalue of matrix $M_1 - \eta M_2$ with a positive eigenvector v. Thus, the comparison principle [40, Theorem B.1] implies that for sufficiently small $\bar{\varepsilon} > 0$ and any initial value $(E(0), I(0))^T$ with $(E(0), I(0))^T \geq \bar{\varepsilon}v$, we have

$$(E(t), I(t))^T \geq \bar{\varepsilon} v e^{s(M_1 - \eta M_2)t}, \ \forall t \geq 0.$$

Therefore, $E_i(t) \to \infty, I_i(t) \to \infty$ as $t \to \infty, i \in \{r, m, u\}$. It then follows that inequality (12) holds.

In view of the above-mentioned claim, P_0 is an isolated invariant set in X and $W^s(P_0) \cap X_0 = \phi$. Further, every orbit in M_∂ approaches P_0, and P_0 is acyclic in M_∂. By the continuous-time version of [48, Theorem 1.3.1 and Remark 1.3.1], there is some $\zeta > 0$ so that $\liminf_{t\to\infty} d(\Phi_t(x), \partial X_0) > \zeta, \forall x \in X_0$. Thus, for the $\zeta > 0$, the solution $(S(t), E(t), I(t))$ associated with system (3) with the initial value $(S(0), E(0), I(0)) \in X_0$ satisfies

$$\liminf_{t\to\infty} E_i(t) > \zeta, \quad \liminf_{t\to\infty} I_i(t) > \zeta, \quad i \in \{r, m, u\},$$

that is, Φ_t is uniformly persistent with respect to X_0. Since the semiflow $\Phi_t : X \to X$ is point dissipative and compact for each $t > 0$, [48, Theorem 1.3.7] implies that system (3) has at least one equilibrium $(S^{**}, E^{**}, I^{**}) \in X_0$. Clearly, $E^{**} > 0$, and $I^{**} > 0$. Suppose, by contradiction, that $S_r^{**} = 0$. Then from the first equation of system (3), we get $0 = \Lambda_r + a_{rm}S_m^{**} \geq \Lambda_r > 0$. Thus, we have $S_r^{**} > 0$. By similar contradiction arguments, we can further prove $S_m^{**} > 0$ and $S_u^{**} > 0$. Therefore, $(S^{**}, E^{**}, I^{**}) \in \text{Int}(R_+^9)$. □

In the case where $a_{rm} = b_{rm} = c_{rm} = c_{mr} = 0$, our model system (3) reduces to the model with immigration and cross-infection, which was analyzed in [22]. The only difference is that we consider the treatment of the infectious individuals. According to [22], we can define the basic reproduction number for the rural population as

$$\mathcal{R}_{0r} = \frac{\beta_{rr}}{\mu + \alpha_r + \gamma_r} \frac{\Lambda_r}{\mu + a_{mr}} \left(p_r + (1 - p_r) \frac{k_r}{\mu + k_r + b_{mr}} \right).$$

If $\mathcal{R}_{0r} < 1$, there is another basic reproduction number for the migrant population and urban population $\mathcal{R}_{0mu} = \rho(F_2 V_2^{-1})$, where

$$F_2 = \begin{bmatrix} 0 & 0 & (1-p_m)\beta_{mm}\ddot{S}_m^* & (1-p_m)\beta_{mu}\ddot{S}_m^* \\ 0 & 0 & (1-p_u)\beta_{um}\ddot{S}_u^* & (1-p_u)\beta_{uu}\ddot{S}_u^* \\ 0 & 0 & p_m\beta_{mm}\ddot{S}_m^* & p_m\beta_{mu}\ddot{S}_m^* \\ 0 & 0 & p_u\beta_{um}\ddot{S}_u^* & p_u\beta_{uu}\ddot{S}_u^* \end{bmatrix},$$

$$V_2 = \begin{bmatrix} \mu + k_m & 0 & 0 & 0 \\ 0 & \mu + k_u & 0 & 0 \\ -k_m & 0 & \mu + \alpha_m + \gamma_m & 0 \\ 0 & -k_u & 0 & \mu + \alpha_u + \gamma_u \end{bmatrix},$$

$\ddot{S}_m^* = \frac{a_{mr}\Lambda_r}{\mu(\mu + a_{mr})}$, and $\ddot{S}_u^* = \Lambda_u/\mu$.

If we define $\hat{\mathcal{R}}_0 = \max\{\mathcal{R}_{0r}, \mathcal{R}_{0mu}\}$ as the basic reproduction number for the whole population, the direct calculation shows that if $a_{rm} = b_{rm} = c_{rm} = c_{mr} = 0$, then $\mathcal{R}_0 \equiv \hat{\mathcal{R}}_0$. By the main theorem in [22], we have the following result.

Lemma 3.2. *Assume $a_{rm} = b_{rm} = c_{rm} = c_{mr} = 0$. If $\mathcal{R}_0 < 1$, the system (3) has only one disease-free equilibrium which is globally asymptotically stable; while if $\mathcal{R}_0 > 1$, the system (3) has a unique endemic equilibrium $(\hat{S}^{**}, \hat{E}^{**}, \hat{I}^{**})$ which is globally asymptotically stable.*

Theorem 3.3. *Let $\Lambda = R_+^4$, $\lambda = (a_{rm}, b_{rm}, c_{rm}, c_{mr}) \in \Lambda$, $\lambda_0 = (0, 0, 0, 0)$, and $\mathcal{R}_{0\lambda}$ be the basic reproduction number of system (3) with parameter λ. If $\mathcal{R}_0 > 1$, then there exists an $\hat{\varepsilon} > 0$ such that for any $\lambda \in \Lambda$ satisfying $||\lambda - \lambda_0|| \leq \hat{\varepsilon}$, system (3) admits a unique endemic equilibrium $(S_\lambda^{**}, E_\lambda^{**}, I_\lambda^{**})$ such that $\lim_{t\to\infty}(S(t) - S_\lambda^{**}) = 0$, $\lim_{t\to\infty}(E(t) - E_\lambda^{**}) = 0$, and $\lim_{t\to\infty}(I(t) - I_\lambda^{**}) = 0$ for every $(S(0), E(0), I(0)) \in X_0$.*

Proof. Since $\mathcal{R}_0 > 1$, there exists an $\hat{\varepsilon}_0 > 0$ such that $\mathcal{R}_{0\lambda} > 1$ for each $\lambda \in \Lambda$ with $||\lambda - \lambda_0|| \leq \hat{\varepsilon}_0$. We denote $\Lambda_0 := \{\lambda \in \Lambda : ||\lambda - \lambda_0|| \leq \hat{\varepsilon}_0\}$. Lemma 2.1 implies that $s(M_1) > 0$. Then there is a sufficiently small $\hat{\delta}_0 > 0$ such that $s(M_1 - \hat{\delta}_0 M_2) > 0$. The similar proof of Theorem 3.1 implies that there exists some $\hat{\zeta}_0 > 0$ and $\hat{\varepsilon}_1 \in (0, \hat{\varepsilon}_0]$ such that

$$\limsup_{t\to\infty} d(\Phi_t^\lambda(x), P_0) \geq \hat{\zeta}_0, \quad \forall x \in X_0,\ \lambda \in \Lambda_1 := \{\lambda \in \Lambda : ||\lambda - \lambda_0|| \leq \hat{\varepsilon}_1\},$$

where $\Phi_t^\lambda : X \to X$ is the solution semiflow of system (3) with parameter λ.

Note that $\Phi_t^\lambda(X_0) \subset X_0$ for all $t \geq 0$ and $\lambda \in \Lambda_1$. It is easy to see that solutions of system (3) in X are uniformly bounded and ultimately bounded for each $\lambda \in \Lambda_1$. Thus, Φ_t^λ admits a global attractor $A_\lambda \subset X_0$ for each $\lambda \in \Lambda_1$. By the continuous-time version of [48, Theorem 1.4.2] on uniform persistence uniform in parameters, there exists some $\hat{\zeta}_1 \in (0, \hat{\zeta}_0]$ and $\hat{\varepsilon}_2 \in (0, \hat{\varepsilon}_1]$ such that

$$\liminf_{t\to\infty} d(\Phi_t^\lambda(x), \partial X_0) \geq \hat{\zeta}_1,\ \forall x \in X_0,\ \lambda \in \Lambda_2 := \{\lambda \in \Lambda : ||\lambda - \lambda_0|| \leq \hat{\varepsilon}_2\}.$$

Clearly, the ultimate boundedness of solutions of system (3) implies that there is a bounded and closed set $\Omega^* \subset X_0$ such that $A_\lambda \subset \Omega^*$, $\forall \lambda \in \Lambda_2$. Since $\overline{\cup_{\lambda\in\Lambda_2}\Phi_t^\lambda(A_\lambda)} = \overline{\cup_{\lambda\in\Lambda_2}A_\lambda} \subset \overline{\Omega^*} = \Omega^* \subset X_0$, $\overline{\cup_{\lambda\in\Lambda_2}\Phi_t^\lambda(A_\lambda)}$ is compact in X_0. Lemma 3.2 implies that when $\lambda = 0$, system (3) admits a unique endemic equilibrium $(\hat{S}^{**}, \hat{E}^{**}, \hat{I}^{**})$ which is globally asymptotically stable in X_0. By the continuous-time version of [48, Theorem 1.4.1], there is an $\hat{\varepsilon}_3 \in (0, \hat{\varepsilon}_2]$ such that for each $\lambda \in \Lambda$ with $||\lambda - \lambda_0|| \leq \hat{\varepsilon}_3$, system (3) has a unique endemic equilibrium $(S_\lambda^{**}, E_\lambda^{**}, I_\lambda^{**})$ with $(S_0^{**}, E_0^{**}, I_0^{**}) = (\hat{S}^{**}, \hat{E}^{**}, \hat{I}^{**})$, and $(S_\lambda^{**}, E_\lambda^{**}, I_\lambda^{**})$ is globally attractive in X_0. □

Theorems 3.1 and 3.3 imply that the basic reproduction number $\mathcal{R}_0$ plays an important role for model (1) and is a threshold parameter to determine the persistence or eradication of TB. Fig. 2 shows such a case where there exists a unique global attractive endemic equilibrium when $\mathcal{R}_0 = 1.1910 > 1$. Our intensive simulations in a wide range of plausible parameter ranges show this global attractivity of a unique positive equilibrium.

4. A case study.

In this section, we conduct some numerical simulations based on the available data relevant to the mega city of Beijing and its major sources of migrant workers. In our general analysis of the model (1), we used different migration rates a_{rm}, b_{rm}, and e_{rm}. However, it is difficult to distinguish susceptible migrant workers, exposed/latent migrant workers, and recovered/treated migrant workers, and to control their migration rates between their villages and towns/cities. Thus, in the simulations below, we use $a_{rm} = b_{rm} = e_{rm}$. Similarly, we suppose that $a_{mr} = b_{mr} = e_{mr}$.

4.1. Initial values and model parameters.

We fix the year 2000 as the initial time and the time unit will be one year. From the web site of the National Bureau of Statistic of China [32], the number of urban residents residing in Beijing for longer than six months P_1, the number of permanent urban residents in Beijing P_2, the number of the total migrant workers in China P_3, and the number of the rural population in China P_4 can be obtained from 2000 to 2008 (Table 1). Clearly, the migrant workers in Beijing P_5 corresponds to the difference of urban residents in Beijing longer than six months and permanent urban residents in Beijing. Therefore, the migrant workers in Beijing accounts for 2.913% of the total migrant workers in China. The rural population residing in villages longer than six months during one year P_6 corresponds to the difference of the rural population and the total migrant workers in China. Thus, the number of the rural population residing in villages longer than six months during one year corresponding to the migrant workers P_7 should be the product of the rural population residing in villages longer than six months during one year P_6 and the average fraction of the migrant workers in Beijing in the whole migrant workers in China.

From the web site of the National Bureau of Statistic of China [32], we can get the average birth rates of the whole population B_1 from 2000 to 2008 (see Table 2), therefore, the recruitment rate RR_1 of P_7 is the product of B_1 and P_7 (see Table 2). Thus, the average recruitment rate Λ_r of those nine years is taken to be 237770. By [32], we can obtain the average birth rates B_2 of permanent urban residents in Beijing from 2000 to 2008 (see Table 2), therefore, the recruitment rate RR_2 of P_2 is the product of B_2 and P_2 (see Table 2). Thus, the average recruitment rate Λ_u of those nine years is fixed to be 76691. The average life expectancy of uninfected individuals is 71.4 years and hence $\mu = 1/71.4$ [32].

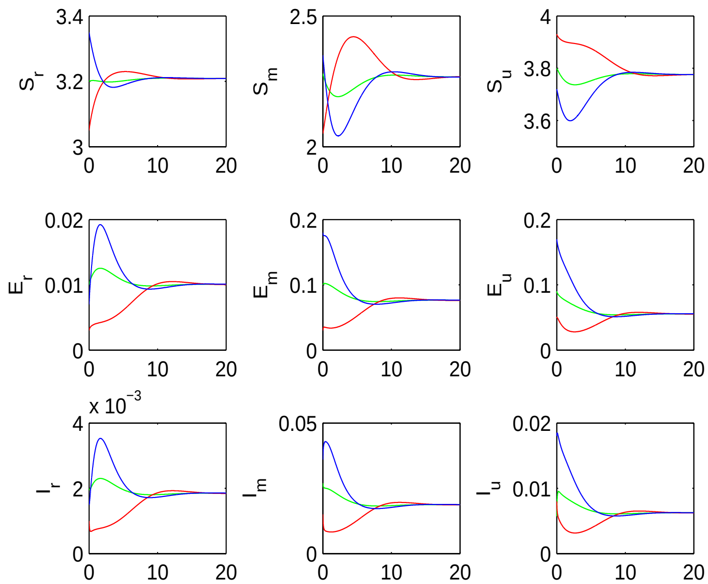

FIGURE 2. Global attractivity of the unique endemic equilibrium of system (3) when $\mathcal{R}_0 = 1.1910$. We choose $\Lambda_r = 3$, $\Lambda_u = 2$, $\mu = 0.5$, $a_{mr} = 0.5$, $a_{rm} = 0.1$, $b_{mr} = 0.4$, $b_{rm} = 0.15$, $c_{mr} = 0.01$, $c_{rm} = 0.09$, $k_r = 2$, $k_m = 2.5$, $k_u = 1.5$, $p_r = 0.02$, $p_m = 0.03$, $p_u = 0.01$, $\alpha_r = 1.5$, $\alpha_m = 2$, $\alpha_u = 1$, $\gamma_r = 10$, $\gamma_m = 8$, $\gamma_u = 12$, $\beta_{rr} = 3.0556$, $\beta_{mm} = 5.5$, $\beta_{mu} = 0.7333$, $\beta_{uu} = 1.75$, and $\beta_{um} = 1$. The three sets of the initial values of $(S_r, S_m, S_u, E_r, E_m, E_u, I_r, I_m, I_u)$ are chosen like these: (3.2, 2.28, 3.8, 0.01, 0.1, 0.09, 0.002, 0.027, 0.0057), (3.05, 2.05, 3.93, 0.003, 0.03, 0.05, 0.001, 0.015, 0.008), and (3.35, 2.35, 3.72, 0.007, 0.18, 0.17, 0.0015, 0.036, 0.018), respectively.

TABLE 1. The numbers of seven kinds of people in China(Unit: thousand)

Year	P_1	P_2	P_3	P_4	P_5	P_6	P_7
2000	1363.6	1107.5	8840	80837	256.1	71997	2097.3
2001	1385.1	1122.3	8961	79563	262.8	70602	2056.6
2002	1423.2	1136.3	10488	78241	286.9	67753	1973.6
2003	1456.4	1148.8	11390	76851	307.6	65461	1906.9
2004	1492.7	1162.9	11823	75705	329.8	63882	1860.9
2005	1538	1180.7	12473.3	74544	357.3	62071	1808.1
2006	1581	1197.6	13181	73742	383.4	60561	1764.1
2007	1633	1213.3	13611	72750	419.7	59139	1722.7
2008	1695	1229.9	14041	72135	465.1	58094	1692.3

TABLE 2. The birth rates and the recruitment rates

Year	B_1	RR_1	B_2	RR_2
2000	0.01403	294250	0.006	66450
2001	0.01338	275180	0.0061	68460
2002	0.01286	253810	0.0066	75000
2003	0.01241	236640	0.0051	58590
2004	0.01229	228700	0.0061	70940
2005	0.01240	224210	0.0063	74380
2006	0.01209	213280	0.00626	74970
2007	0.01210	208450	0.00832	10095
2008	0.01214	205440	0.00817	10048

From [30], in 2000, the detection rate of infectious TB cases was 41.4%, 98.9% of detected infectious TB cases were treated, and the normal treatment rate was 27.3%. If the DOTS strategy is used to treat the infectious TB cases, the infectious period of infectious TB cases is thought of as about two months. Thus, approximately, we have

$$\gamma_r = \gamma_u = \frac{12}{2}\times 41.4\%\times 98.9\%\times 27.3\%\simeq 0.6707.$$

Note that the infectious migrant workers may not be able to get treated at all in towns/cities unless they return to their home villages, because subsidized management of tuberculosis is only available through facilities in the area where they were registered at birth [25, 38]. The infectious migrant workers return to their homes to get treated only when they are detected. For the infectious migrant workers who do not return to homes, the removal rate is thought of as self-removal rate but not because of getting antituberculosis drugs. The infectious period of infectious migrant workers is five years [14] and $\gamma_m = 0.2$.

Adding the first four equations of system (1) for rural residents and the second four equations of system (1) for migrant workers, we get the following system:

$$\begin{aligned}\frac{dN_r}{dt} &= \Lambda_r + a_{rm}N_m - (\mu + a_{mr})N_r + (a_{mr} - c_{mr} - \alpha_r)I_r + (c_{rm} - a_{rm})I_m,\\ \frac{dN_m}{dt} &= a_{mr}N_r - (\mu + a_{rm})N_m + (a_{rm} - c_{rm} - \alpha_m)I_m + (c_{mr} - a_{mr})I_r.\end{aligned}$$

To get the estimates of a_{mr}, and a_{rm}, we drop out the terms involving I_r and I_m (infectious TB cases account for a very small proportion of the rural population). We then have the following system:

$$\begin{aligned}\frac{dN_r}{dt} &\simeq \Lambda_r + a_{rm}N_m - (\mu + a_{mr})N_r,\\ \frac{dN_m}{dt} &\simeq a_{mr}N_r - (\mu + a_{rm})N_m.\end{aligned}$$

By the least square method, fitting the rural residents corresponding to the migrant workers and the total migrant workers data, respectively (see Fig. 3), we derive that $N_r(0) = 20603000$, $N_m(0) = 1878400$, $a_{mr} = 0.0231$, and $a_{rm} = 0.005$. We assume that the migration rate of infectious migrant workers moving from their home villages to towns/cities is very small, $c_{mr} = 0.00231$, and the migration rate of infectious migrant workers moving from towns/cities to their home villages is larger, $c_{rm} = 0.05$.

From [24, 47], the numbers of infectious migrant workers and infectious urban residents in Beijing can be summarized in Table 3. By applying the known parameter values and the least square method to fit the data of infectious migrant workers and infectious urban residents in Beijing, we can get the other initial values and

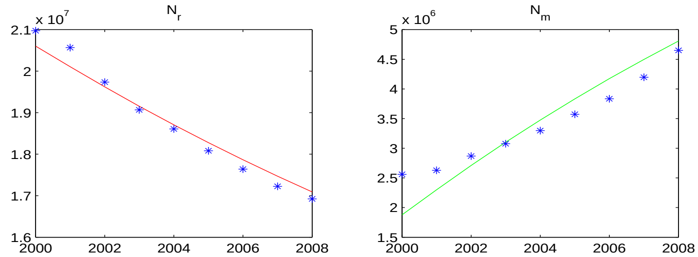

FIGURE 3. The fitted curves of rural population and migrant workers.

TABLE 3. The numbers of TB cases registered in Beijing from 2000 to 2006

Year	2000	2001	2002	2003	2004	2005	2006
Infectious migrant workers	819	923	1075	1168	1387	1266	1638
Infectious permanent residents	2312	2155	2204	2102	2437	2328	2450

TABLE 4. The initial values of the model.

Symbol	Interpretation	Value (persons)
$S_r(0)$	Initial value of susceptible rural residents	15300000
$S_m(0)$	Initial value of susceptible migrant workers	2304900
$S_u(0)$	Initial value of susceptible urban population	9900000
$E_r(0)$	Initial value of exposed/latent rural residents	1700000
$E_m(0)$	Initial value of exposed/latent migrant workers	100334
$E_u(0)$	Initial value of exposed/latent urban population	614580
$I_r(0)$	Initial value of infectious rural residents	2996
$I_m(0)$	Initial value of infectious migrant workers	887
$I_u(0)$	Initial value of infectious urban population	2043

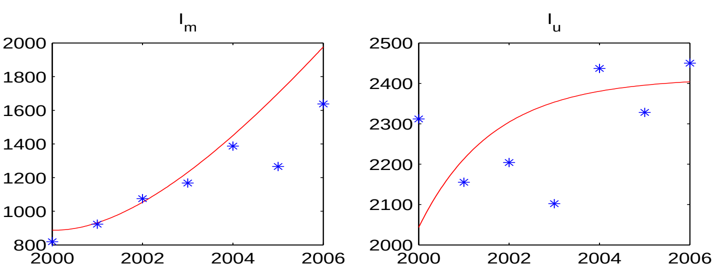

FIGURE 4. The yearly numbers of infectious migrant workers and infectious urban residents (the blue stars) and their fitted curves.

parameter values, which are summarized in Table 4 and Table 5, respectively. The fitted curves are seen in Fig. 4.

Notice that if the governments encourage more and more migrant workers to go back home, and provide them opportunities to facilitate their working at their

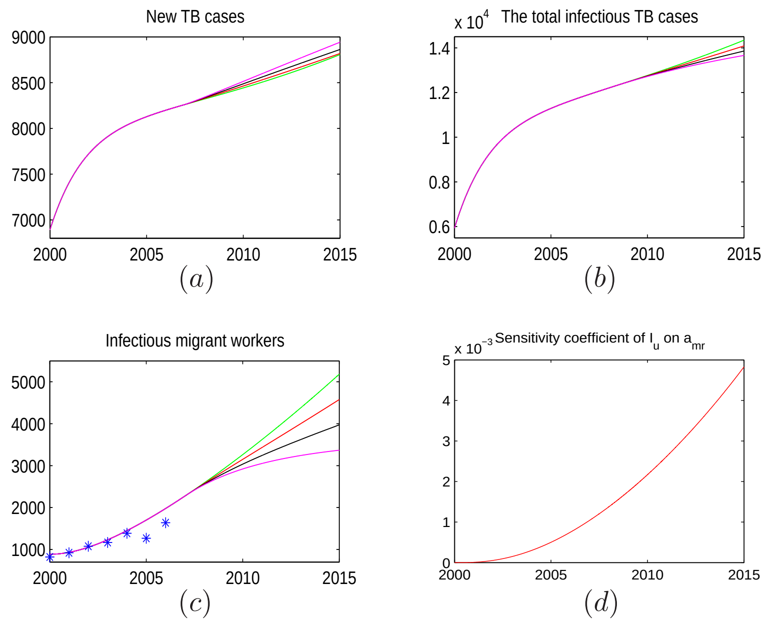

FIGURE 5. Simulations about the impact of reducing the migration rates from the villages to towns/cities. In the first three panels, the green curve corresponds to $a_{mr} = 0.0231$, the red curve corresponds to $a_{mr} = 0.01617$, the black curve corresponds to $a_{mr} = 0.00924$, and the pink curve corresponds to $a_{mr} = 0.00231$.

villages, a_{mr} will rapidly get smaller and a_{rm} will be substantially bigger. If governments and/or companies improve the migrant workers' housing and living conditions, provide migrant workers free health examination periodically, and/or treat exposed/latent migrant workers with antituberculosis drugs, the reactivation rate k_m of exposed/latent migrant workers will become lower. Otherwise, the reactivation rate of migrant workers may remain high. Thus, a_{mr}, a_{rm}, and k_m are thought of as control parameters and are used as varying parameters in the following simulations.

4.2. **Control measures.**

4.2.1. *Building a new type of countryside.* If the governments make greater efforts to educate and train farmers and to facilitate their working at their home villages, there may be less and less migrant workers to leave the countryside for the wage economy in towns/cities [38]. As a consequence, the parameter a_{mr} can thus be reduced, and correspondingly a_{rm} can be increased.

Fig. 5 and Fig. 6 show the impacts of these measures in reducing a_{mr} and increasing a_{rm} on the numbers of new TB cases, the total infectious cases, infectious migrant workers, and infectious permanent urban residents, respectively. From the first three panels of Fig. 5, the numbers of total infectious cases and infectious migrant workers may be substantially decreased as a_{mr} is decreased. However, the

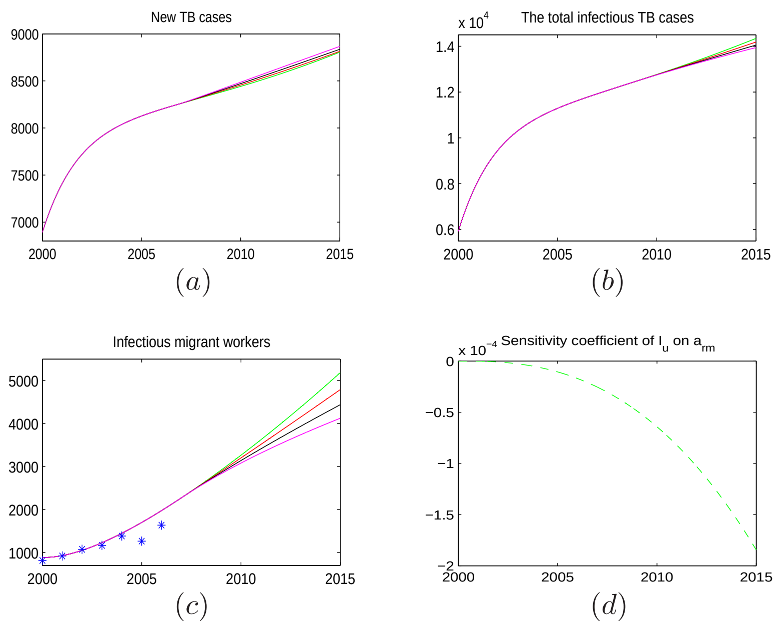

FIGURE 6. Simulations about the impact of increasing the rates of migrants returning to their villages. The green, red, black, and pink curves are correspondingly to a_{rm} =0.005, 0.02, 0.035, and 0.05, respectively in the first three panels.

number of new TB cases may be increased as a_{mr} is decreased. If a_{mr} is decreasing, there will be less and less migrant workers working in the towns/cities, resulting in the reduction of the number of the total infectious cases and infectious migrant workers. The numbers of new TB cases, total infectious cases and infectious migrant workers may increase over time. The trends of infectious urban population as a_{mr} changes are addressed by our consideration of the sensitivity coefficient (some details about sensitivity coefficients can be seen in subsection 4.3) of infectious urban population on a_{mr} because the number of infectious urban population has a very small change as a_{mr} decreases. From Fig. 5d, the sensitivity coefficient of infectious urban population will increase over time.

Fig. 6 also indicates that the change of a_{rm} yields great effects on new TB cases, total infectious cases, infectious migrant workers, and infectious urban population. The numbers of total infectious cases and infectious migrant workers may decrease, while the number of new TB cases is increasing as a_{rm} increases. If a_{rm} increases, there are more and more migrant workers to return to their villages again. Thus, the numbers of total infectious cases and infectious migrant workers may decrease. From Fig. 6, the sensitivity coefficient of infectious urban population is decreasing over time, but a_{rm} has just minor influence on the infectious urban population.

4.2.2. *More attention to migrant workers.* Despite the current huge progress of Chinese health-system reforms in tuberculosis control, the issue of migrant workers has not received its deserved attention [38, 43]. The migrant workers always live and

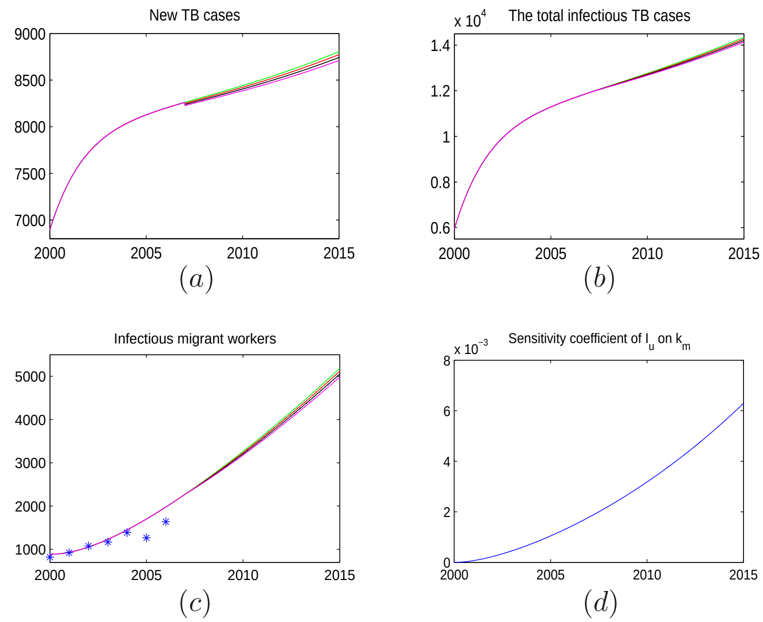

FIGURE 7. Simulations about the impact of decreasing the reactivation rate of exposed/latent migrant workers. k_m is replaced, from top to bottom, by 0.0025, 0.002467, 0.002434, and 0.0024 in the first three panels.

work in those environments that promote transmission of tuberculosis and impede effective diagnosis and treatment [38]. Further, migrant workers have no access to treatment in towns/cities as they have to return home for treatment if they get tuberculosis [38]. A better prevention and treatment program for migrant workers will benefit the whole society, not only because migrant workers contribute to the growing economy but also they are exposed to the great risk of TB infection and they can pass on the infection to the entire population.

If governments at all levels and/or companies take some actions such as free access to regular health examination, improvement of living and working conditions, and speedy treatment, there will be less exposed migrant workers to progress quickly to infectious TB cases and/or newly infected migrant workers. If some actions are taken, k_m will be reduced. From the first three panels of Fig. 7, the numbers of new TB cases, total infectious TB cases, and infectious migrant workers will dramatically decrease as k_m is becoming smaller and smaller.

4.3. **Sensitivity analysis.** Sensitivity analysis of parameters is not only critical to model verification and validation in the process of model development and refinement, but also provides insight to the robustness of model results when making decision [35]. We use sensitivity coefficient to show sensitivity analysis. The sensitivity coefficient SC_{Yy} of some variable Y on parameter y can be defined as the difference of Y divided by the difference of y. If SC_{Yy} is positive, $\triangle Y$ ($\triangle Y$ is the difference of Y) and $\triangle y$ share the same change direction; while if SC_{Yy} is negative,

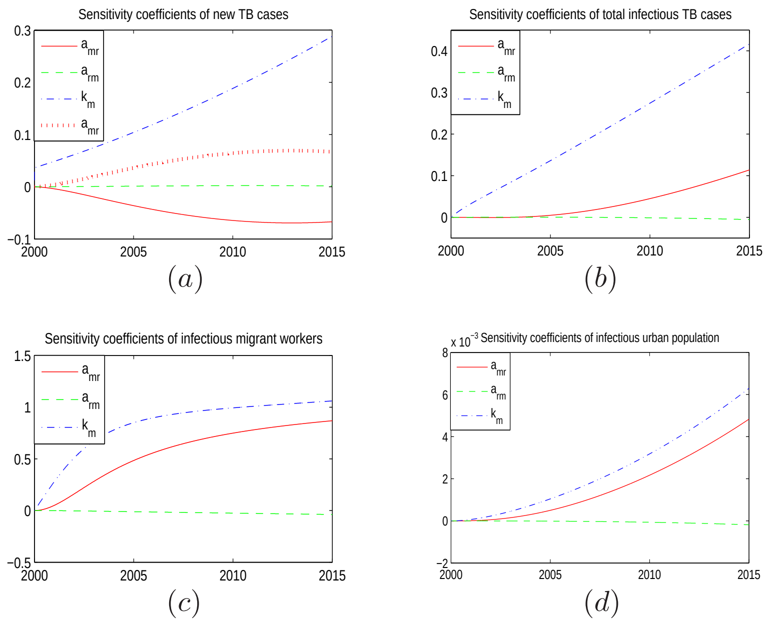

FIGURE 8. The comparison of sensitivity coefficients.

they have the opposite change direction. The bigger the absolute values of SC_{Yy}, the greater the impact of parameter y on the variable Y. Thus, parameter y plays an important role in changing Y. The sensitivity coefficient of Y on parameter y can be also interpreted as the percentage change in the number of Y for 1% change in the parameter y.

Fig. 8 gives sensitivity coefficients of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population) on parameters a_{mr}, a_{rm}, and k_m, respectively. The red curve, the green curve, and the blue curve correspond to a_{mr}, a_{rm}, and k_m, respectively. And the red dotted curve implies the absolute value of sensitivity coefficient of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population) on parameters a_{mr} changes over time.

Parameters k_m has the greatest influence on new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population) and the number of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population) is most sensitive to parameters k_m. As such, k_m is the most sensitive parameter and plays the most important role in determining the number of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population). a_{mr} is more sensitive than a_{rm}, in deciding the number of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population). We conclude that more special attention should be paid to reducing the reactivation rate k_m of exposed migrant workers or the migrant rate a_{mr}. Effective actions should be taken to slow down the progress of exposed/latent migrant workers to infectious TB cases, and more farmers should be encouraged stay at their villages.

Table 5. The parameter values used in the simulations.

Symbol	Value	Interpretation	References
Λ_r	237770	Recruitment rate in the rural areas	See text
Λ_u	76691	Recruitment rate in the urban areas	See text
μ	1/71.4	Natural death rate	[32]
a_{mr}	0.0231	Migration rate of susceptible people from rural areas to urban areas	Fit
a_{rm}	0.005	Migration rate of susceptible people from urban areas to rural areas	Fit
b_{mr}	0.0231	Migration rate of latent people from rural areas to urban areas	Fit
b_{rm}	0.005	Migration rate of latent people from urban areas to rural areas	Fit
c_{mr}	0.00231	Migration rate of infectious people from rural areas to urban areas	See text
c_{rm}	0.05	Migration rate of infectious people from urban areas to rural areas	See text
e_{mr}	0.0231	Migration rate of removed people from rural areas to urban areas	Fit
e_{rm}	0.005	Migration rate of removed people from urban areas to rural areas	Fit
k_r	0.0024	Reactivation rate of exposed/latent rural residents	Fit
k_m	0.0025	Reactivation rate of exposed/latent migrant workers	Fit
k_u	0.0024	Reactivation rate of exposed/latent urban population	Fit
p_r	0.03	Fraction of new infections that become TB disease in rural areas	Fit
p_m	0.035	Fraction of new infections that become TB disease in migrant workers	Fit
p_u	0.028	Fraction of new infections that become TB disease in urban areas	Fit
α_r	0.075	Disease-induced death rate of the infectious rural residents	Fit
α_m	0.0806	Disease-induced death rate of the infectious migrant workers	Fit
α_u	0.065	Disease-induced death rate of the infectious urban population	Fit
γ_r	0.6707	Removal rate of infectious rural residents	See text
γ_m	0.2	Removal rate of infectious migrant workers	See text
γ_u	0.6707	Removal rate of infectious urban population	See text
β_{rr}	5.501×10^{-7}	Transmission coefficient from infectious rural residents to susceptible rural population	Fit
β_{mm}	5.9172×10^{-7}	Transmission coefficient from infectious migrant workers to susceptible migrant workers	Fit
β_{uu}	5×10^{-7}	Transmission coefficient from infectious urban residents to susceptible urban population	Fit
β_{mu}	10^{-8}	Transmission coefficient from infectious urban population to susceptible migrant workers	Fit
β_{um}	5×10^{-9}	Transmission coefficient from infectious migrant workers to susceptible urban population	Fit

5. **Discussion.** In the present paper, a TB model incorporating migration was developed. The whole population is classified into three subgroups, rural population, migrant workers, and urban population. In each subgroup, there are four classes of subpopulation depending on their disease status. The basic reproduction number $\mathcal{R}_0$ is given according to [42]. It is shown that TB disease dies out and model (1) has only one disease-free equilibrium which is globally asymptotically stable if $\mathcal{R}_0<1$;

while TB disease persists in the population and model (1) has at lease one endemic equilibrium if $\mathcal{R}_0 > 1$. Furthermore, if the migration rates of migrant workers from villages to towns/cities and infectious migrant workers from towns/cities to villages are very small, model (1) has exactly one endemic equilibrium which is globally attractive provided $\mathcal{R}_0 > 1$. Numerical simulation (Fig. 2) suggests that there is a globally stable endemic equilibrium for all parameter values when $\mathcal{R}_0 > 1$.

If governments and/or companies effectively improve migrant workers' housing and living conditions, substantially treat exposed/latent migrant workers by providing them free antituberculosis drugs such as INH, and/or provide migrant workers free health examination in order to slow down their reactivation to active TB cases, the spread of TB may be dramatically lowered. Propagandizing the knowledge about TB to migrant workers, encouraging migrant workers stay at home for a new countryside economy, and providing them more technological knowledge and funds to construct their new villages may be substantially helpful to reduce the number of new TB cases (infectious TB cases, infectious migrant workers, or infectious urban population).

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