

COMPLEX DYNAMICS OF A SIMPLE EPIDEMIC MODEL WITH A NONLINEAR INCIDENCE

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ABSTRACT. A simple epidemic model with a nonlinear incidence rate and two compartments is studied. The backward bifurcation is described and the corresponding threshold is calculated. The Hopf bifurcation and Bogdanov-Takens bifurcation are analyzed and numerical evidences for the stable or unstable limit cycle are provided.

1. Introduction. The importance of the so-called basic reproductive number, R_0 , has been well recognized in the theoretical study of disease spread. In particular, for many deterministic models, R_0 classifies the long-term progression of the disease under consideration. Namely, if $R_0 < 1$, then the disease-free equilibrium is globally stable; if $R_0 > 1$, then a unique endemic equilibrium appears and this endemic equilibrium is globally stable.

However, there are increasing evidences indicating that R_0 alone cannot fully determine the global dynamics of the disease transmission. Indeed, backward and/or Hopf bifurcations can happen [6, 7, 15, 11, 9, 10, 14, 13, 12, 16, 17, 19] and the Bogdanov-Takens singularity may occur [18]. These complicated dynamics can be the consequence of non-standard incidence rates [3, 13, 16, 17]. Multiple equilibria and periodic solutions are shown to exist in [13, 16, 17], and multiple limit cycles are observed in [18] for a simple SI-model with the incidence rate $\frac{kI^2S}{1+\alpha I^2}$ (with parameters $k, \alpha > 0$).

In this paper, we consider a simple SI -epidemic model with a constant recruitment rate and the incidence rate $\beta(1 + \epsilon I)IS$ suggested in [19, 5, 1]. We show that this simple model can exhibit very complicated dynamical behaviors. First

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