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Stability of Neutral Functional Differential Equations with Infinite Delay

By

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§ 1. General theorems

Let C be the space of bounded continuous functions mapping $(-\infty, 0]$ to $\boldsymbol{R}^n$, C^1 be a subspace of C, the elements of which have bounded continuous derivative. For $\varphi \in C$, we define the norm $\|\varphi\| = \sup_{s\leqslant 0}|\varphi(s)|$, where $|\cdot|$ is the Euclidean norm in $\boldsymbol{R}^n$. $C_H = \{\varphi \in C;\ \|\varphi\| < H\}$, $C_H^1 = \{\varphi \in C_H \cap C^1;\ \dot{\varphi} \in C_H\}$. For $A > 0$, $t_0 \in \boldsymbol{R}$, x: $(-\infty, t_0+A) \to \boldsymbol{R}^n$, $t \in [t_0, t_0+A]$ define x_t: $[-\infty, 0] \to \boldsymbol{R}^n$ as $x_t(\theta) = x(t+\theta)$ for $\theta \leqslant 0$.

Consider the neutral functional differential equation

$$(1) \qquad \dot{x}(t) = f(t, x_t, \dot{x}_t) \qquad t \geqslant t_0$$

where f: $\boldsymbol{R} \times C_H \times C_H \to \boldsymbol{R}^n$ is continuous, $f(t, 0, 0) \equiv 0$. Throughout this paper, we always suppose the solution $x(t; t_0, \varphi)$ of (1) through $(t_0, \varphi) \in \boldsymbol{R} \times C_H^1$ exists and is unique.

Definition 1. We say the zero solution of (1) is stable if for any $\varepsilon > 0$, $t_0 \in R$, there exists a $\delta(\varepsilon, t_0) > 0$ such that $|x(t; t_0, \varphi)| < \varepsilon$ for any $\varphi \in C_\delta^1$ and $t \geqslant t_0$.

If δ is independent of t_0, then we say the zero solution of (1) is uniformly stable.

If the zero solution of (1) is stable and there is a $\delta(t_0) > 0$ such that for any $\varphi \in C_{\delta(t_0)}^1$, $\lim_{t\to\infty} x(t; t_0, \varphi) = 0$, then we say the zero solution of (1) is asymptotically stable.

Condition 1. There exist nonnegative continuous non-decreasing functions k_1, k_2 with $k_1(0) = k_2(0) = 0$ such that

$$|f(t, x_t, \dot{x}_{(t)})| \leqslant k_1(\|x_t\|) + k_2(\|\dot{x}_t\|).$$

Condition 2. If $z \leqslant k_1(\sigma) + k_2(z)$, then $z \leqslant k(\sigma)$, where k is a continuous, strictly increasing function with $k(s) > 0$ for $s > 0$.

Condition 3. There exists functions p: $\boldsymbol{R}^+ \to \boldsymbol{R}$, F: $\boldsymbol{R} \times \boldsymbol{R}^+ \times \boldsymbol{R}^+ \to \boldsymbol{R}^+$ such that

$$p(t) \leqslant t, \quad p(t) \to \infty \quad \text{as} \quad t \to \infty$$

and for any $M > 0$, $\sup_{0 \leqslant x, y \leqslant M} F(t, x, y) \to 0$ as $t \to \infty$,

$$|f(t, x_t, \dot{x}_t)| \leqslant k_1(\sup_{p(t)\leqslant s\leqslant t} |x(s)|) + k_2(\sup_{p(t)\leqslant s\leqslant t} |\dot{x}(s)|) + F(t, \|x_t\|, \|\dot{x}_t\|)$$

where k_1, k_2 is defined as condition 1.

Condition 4. i) There exists a continuous V: $\boldsymbol{R}\times\boldsymbol{R}^n\to\boldsymbol{R}^+$ such that

$$u(|x|) \leqslant V(t, x) \leqslant w(|x|)$$

where u, w are nonnegative and continuous, with $u(0)=w(0)=0$, u is strictly increasing and w nondecreasing.

ii) When $V(s, x(s)) \leqslant V(t, x(t))$, $|\dot{x}(s)| \leqslant k(u^{-1}(V(t, x(t))))$ hold for $s \leqslant t$,

$$\dot{V}_{(1)}(t, x(t)) \leqslant W_1(t, V(t, x(t)))$$

where W_1: $\boldsymbol{R}\times\boldsymbol{R}^+\to\boldsymbol{R}^+$ is continuous and the zero solution of

$$(2) \qquad \dot{y}(t) = W_1(t, y(t))$$

is uniformly stable.

Condition 5. (i) The same as (i) of Condition 4.

(ii) The exists a function p: $\boldsymbol{R}^+\to\boldsymbol{R}$ such that $p(t)\leqslant t$ and $p(t)\to\infty$ as $t\to\infty$ and when

$$V(s, x(s)) \leqslant N, \qquad |\dot{x}(s)| \leqslant k(u^{-1}(N))$$

for an arbitrary N and all $s \in [p(t), t]$,

$$\dot{V}_{(1)}(t, x(t)) \leqslant F(t, V, N)$$

where

$$F(t, V, V) \leqslant -W_2(V) + g_1(t)G(V) + g_2(t)Q(V)$$

$W_2(r)$ is continuous and positive for $r>0$; $g_1(t)\geqslant 0$, $\int_0^{+\infty} g_1(s)ds < +\infty$, $G(V)$, $Q(V)$ $\geqslant 0$ are continuous, $g_2(t)\geqslant 0$ and $g_2(t)\to 0$ as $t\to\infty$.

$$|F(t, V, N_1) - F(t, V, N_2)| \leqslant [h_1(t) + h_2]|N_1 - N_2|$$

$h_1(t)\geqslant 0$, $\int_0^{+\infty} h_1(t)dt < +\infty$, h_2 is a nonnegative constant.

Theorem 1. *If the Conditions* 1, 2, 4 *hold, then the zero solution of equation* (1) *is uniformly stable.*

Proof. For any $\varepsilon>0$, by the uniform stability of the zero solution of equation (2), there exists $y_0(\varepsilon)>0$, such that for any $t_0\geqslant 0$, the maximal solution $y(t; t_0, y_0(\varepsilon))$ of (2) through $(t_0, y_0(\varepsilon))$ satisfies

$$y(t;\, t_0,\, y_0(\varepsilon)) < u(\varepsilon) \qquad \text{for } t \geqslant t_0.$$

Chosing $\delta(\varepsilon) > 0$ such that

$$w(\delta) < y_0(\varepsilon) \quad \text{and} \quad \delta < k(u^{-1}(y_0(\varepsilon))),$$

we shall prove that for any $\varphi \in C^1_\delta$,

$$V(t) \overset{\text{def}}{=} V(t,\, x(t;\, t_0,\, \varphi)) \leqslant y(t;\, t_0,\, y_0(\varepsilon)) \overset{\text{def}}{=} y(t).$$

Let $y_m(t)$ be the solution of the following initial-value problem

$$\begin{cases} \dot{y}_m(t) = W_1(t,\, y_m(t)) + \dfrac{1}{m} & t \geqslant t_0 \\ y_m(t_0) = y_0 \end{cases}$$

then

$$y(t) = \lim_{m \to \infty} y_m(t).$$

Therefore, to prove $V(t) \leqslant y(t)$, it is sufficient to prove $V(t) \leqslant y_m(t)$ for $m = 1, 2, \cdots;\ t \geqslant t_0$.

Assume, on the contrary, there exist a positive integer m and $t_1 > t_0$ such that $V(t_1) = y_m(t_1)$, $V(t) \leqslant y_m(t)$ for $t_0 \leqslant s < t_1$ and there exists a sequence $T_k > t_1$ such that

$$V(T_k) > y_m(T_k) \qquad \text{for } k = 1, 2, \cdots$$

$T_k \to t_1$ as $k \to \infty$. Thus,

$$\dot{V}(t_1) \geqslant \dot{y}_m(t_1) = W_1(t,\, y_m(t_1)) + \frac{1}{m}. \tag{3}$$

Obviously,

$$\begin{aligned} &V(s) \leqslant w(|x(s)|) \leqslant w(\|\varphi\|) \leqslant w(\delta) < y_0 < y_m(t_1) < V(t_1) \qquad \text{for } s \leqslant t_0, \\ &V(s) \leqslant y_m(s) \leqslant y_m(t_1) = V(t_1) \qquad \text{for } t_0 \leqslant s < t_1, \\ &|\dot{x}(s)| \leqslant \delta < k(u^{-1}(y_0)) \leqslant k\,(u^{+}(V(t_1)) \qquad \text{for } s \leqslant t_0. \end{aligned}$$

If $|\dot{x}(t)| = \sup_{s \leqslant t} |\dot{x}(s)|$, $t_0 \leqslant t \leqslant t_1$, then

$$|\dot{x}(t)| \leqslant k_1(\|x_t\|) + k_2(\|\dot{x}_t\|) \leqslant k_1(u^{-1}(V(t_1)) + k_2(|\dot{x}(t)|).$$

From Condition 2, it follows that

$$|\dot{x}(t)| \leqslant k(u^{-1}(V(t_1))).$$

Then, from Condition 4, we get

$$\dot{V}(t_1) \leqslant W_1(t, V(t_1)) = W_1(t, y_m(t_1)),$$

which is contrary to (3).

Therefore, $V(t) \leqslant y_m(t)$ for $t \geqslant t_0$, $m=1, 2, \cdots$, it follows that $V(t) \leqslant y(t) < u(\varepsilon)$, $|x(t)| < \varepsilon$. This completes the proof.

Theorem 2. *If Conditions* 1, 2, 3, 5 *hold and the zero solution of equation* (1) *is stable, then the zero solution of* (1) *is asymptotically stable.*

Proof. For a given positive constant $H_1 < H$, $t_0 \geqslant 0$, by the stability of equation (1), there exists $\delta > 0$ such that for $\varphi \in C_\delta^1$, $t \geqslant t_0$,

$$|x(t; t_0, \varphi)| \leqslant H_1 < H.$$

From Condition 2, we get

$$|\dot{x}(t; t_0, \varphi)| \leqslant \max(\delta, k(H_1)).$$

Let $\overline{\lim}_{t\to\infty} V(t, x(t)) = \bar{\sigma}$, $\underline{\lim}_{t\to\infty} V(t, x(t)) = \underline{\sigma}$, $\overline{\lim}_{t\to\infty} |\dot{x}(t)| = q$ (where $x(t) = x(t; t_0, \varphi)$). For any $\mu > 0$, there exists $\alpha \in (0, \mu)$ such that $k(u^{-1}(\bar{\sigma}+\mu)) \geqslant k(u^{-1}(\bar{\sigma})) + \alpha$. From the definition of upper limit, there exists $T \geqslant t_0$ such that for $t \geqslant T$,

$$V(t, x(t)) \leqslant \bar{\sigma} + \mu$$
$$|\dot{x}(t)| \leqslant q + \alpha \leqslant q + \mu,$$

and hence $|x(t)| \leqslant u^{-1}(\bar{\sigma}+\mu)$.

Since $p(t) \to \infty$ as $t \to \infty$, there is $T_1 \geqslant T$ such that $p(t) \geqslant T$ for $t \geqslant T_1$, and so

$$\sup_{p(t)\leqslant s\leqslant t} |x(s)| \leqslant u^{-1}(\bar{\sigma}+\mu) \qquad \text{for } t \geqslant T_1$$
$$\sup_{p(t)\leqslant s\leqslant t} |\dot{x}(s)| \leqslant q + \alpha \qquad \text{for } t \geqslant T_1.$$

Since $\overline{\lim}_{t\to\infty} |\dot{x}(t)| = q$, we can find a sequence $T_n \geqslant T_1$, $T_n \to \infty$ as $n \to \infty$, such that

$$|\dot{x}(T_n)| \geqslant q - \mu \qquad n = 1, 2, \cdots.$$

By the Condition 3, we get

$$q - \mu \leqslant |\dot{x}(T_n)| \leqslant k_1(u^{-1}(\bar{\sigma}+\mu)) + k_2(q+\mu) + |F(T_n, \|x_{T_n}\|, \|\dot{x}_{T_n}\|)|.$$

Putting $\mu \to 0$, $n \to \infty$, we get

$$q \leqslant k_1(u^{-1}(\bar{\sigma})) + k_2(q).$$

From Condition 2, we have

$$q \leqslant k(u^{-1}(\bar{\sigma})).$$

Thus, for $t \geqslant T_1$, $s \geqslant p(t)$

$$V(s, x(s)) \leqslant \bar{\sigma} + \mu$$
$$|\dot{x}(s)| \leqslant q + \alpha \leqslant k(u^{-1}(\bar{\sigma})) + \alpha \leqslant k(u^{-1}(\bar{\sigma} + \mu)).$$

Then, from Condition 5, we get

$$\begin{aligned}\dot{V}(t) &\leqslant F(t, V(t), \bar{\sigma} + \mu)\\ &\leqslant F(t, V(t), V(t)) + |F(t, V(t), \bar{\sigma} + \mu) - F(t, V(t), V(t))|\\ &\leqslant -W_2(V(t)) + g_1(t)G(V(t)) + g_2(t)Q(V(t)) + [h_1(t) + h_2]|\bar{\sigma} + \mu - V|.\end{aligned}$$

If, $\bar{\sigma} > \underline{\sigma}$, then there exist two sequences k_n, V_n with $k_n \to \infty$ (as $n \to \infty$) and $V_n > 0$, such that

$$V(k_n) = \bar{\sigma} - \mu, \qquad V(k_n + v_n) = \bar{\sigma} - \frac{\mu}{2}$$

$$\bar{\sigma} - \mu \leqslant V(t) \leqslant \bar{\sigma} - \frac{\mu}{2} \qquad k_n \leqslant t \leqslant k_n + V_n.$$

Chosing μ small and n Large enough so that

$$\inf_{\bar{\sigma} - \mu \leqslant s \leqslant \bar{\sigma} - \mu/2} W_2(s) - 2h_2\mu - g_2(t) \sup_{\bar{\sigma} - \mu \leqslant s \leqslant \bar{\sigma} - \mu/2} Q(s) \geqslant 0$$

for $t \in [k_n, k_n + V_n]$, we have

$$\begin{aligned}\dot{V}(t) &\leqslant g_1(t)Q(G(V(t))) + h_1(t)|\bar{\sigma} + \mu - V(t)|\\ &\leqslant g_1(t)M + 2\mu h_1(t) \qquad t \in [k_n, k_n + V_n]\end{aligned}$$

where $G(V(t)) \leqslant M \equiv$ cons t, $t \geqslant t_0$

$$V(k_n + V_n) \leqslant V(k_n) + M \int_{k_n}^{k_n + V_n} g_1(s)ds + 2\mu \int_{k_n}^{k_n + V_n} h_1(s)ds \tag{4}$$

$$\frac{\mu}{2} \leqslant M \int_{k_n}^{k_n + V_n} g_1(s)ds + 2\mu \int_{k_n}^{k_n + V_n} h_1(s)ds.$$

Since $g_1(t)$, $h_1(t) \in L^1[0, \infty)$, we know

$$\lim_{n \to \infty} \int_{k_n}^{k_n + V_n} g_1(s)ds = 0, \qquad \lim_{n \to \infty} \int_{k_n}^{k_n + V_n} h_1(s)ds = 0.$$

Putting $n \to \infty$ in (4), we get a contradiction $\mu \leqslant 0$, which implies $\bar{\sigma} = \underline{\sigma}$.

If $\sigma \overset{\text{def}}{=} \bar{\sigma} = \underline{\sigma} \neq 0$, then there exists $T_2 > T_1$, $\mu > 0$, such that

$$0 < \sigma - \mu < V(t) \leqslant \sigma + \mu, \qquad t \geqslant T_2$$
$$\inf_{\sigma - \mu \leqslant s \leqslant \sigma + \mu} W_2(s) - [2h_2\mu + g_2(t) \sup_{\sigma - \mu \leqslant s \leqslant \sigma + \mu} Q(s)] \geqslant \tfrac{1}{2} W_2(\sigma).$$

Then, for $t \geqslant T_2$,

$$\dot{V}(t) \leqslant -W_2(V(t)) + g_1(t)G(V(t)) + g_2(t)Q(V(t)) + [h_1(t) + h_2]|\sigma + \mu - V(t)|$$
$$\leqslant -W_2(V(t)) + 2h_2\mu + g_2(t)Q(V(t)) + g_1(t)G(V(t)) + 2h_1(t)\mu$$
$$\leqslant -\tfrac{1}{2}W_2(\sigma) + g_1(t)M + 2\mu h_1(t).$$

Hence,

$$V(t) \leqslant V(T_3) - \tfrac{1}{2}W_2(\sigma)(t - T_3) + M\int_{T_3}^{t} g_1(s)ds + 2\mu\int_{T_3}^{t} h_1(s)ds$$

where $t \geqslant T_3 \geqslant T_2$.

From the inequality above and Cauchy theorem, we know if T_3, $t - T_3$ are large enough, then $V(t) < 0$. This, obviously, is a contradiction and then $\sigma = 0$. Then $\lim_{t\to\infty} x(t) = 0$. The proof is completed.

Corollary 1. *If condition* 1–3, 5 *hold and* $g_2(t) \equiv 0$, $\lim_{b\to\sigma^+}\int_b^a 1/G(v)\,dv = \infty$, *for any* $a > 0$, *then the zero solution of* (1) *is asymptotically stable.*

This is an immediate consequence of Theorems 1 and 2.

§ 2. The stability of integrodifferential equations

As the application of the previous theorems, we discuss the stablity of the zero solution of the following integrodifferential equations

$$(5) \qquad \dot{x}(t) = Ax(t) + B_1x(p_1(t)) + B_2\dot{x}(p_2(t)) + \int_{-\infty}^{t} C_1(t-s)x(s)ds + \int_{-\infty}^{t} C_2(t-s)\dot{x}(s)ds,$$

where A, B, B_2 are $n \times n$ constant matrices. $p(t) \leqslant p_1(t)$, $p_2(t) \leqslant t$, $\lim_{t\to\infty} p(t) = \infty$, C_1, C_2 are continuous matrix functions defined on $[0, +\infty]$ and $\int_0^{+\infty} \|C_i(t)\|\,dt < +\infty$ $(i = 1, 2)$.

Suppose A is stable, then there exist a positive definite matrix B and positive constants $\lambda_{\min}$, $\lambda_{\max}$ such that

$$\begin{cases} A^TB + BA = -E \\ \lambda_{\min}|x|^2 \leqslant x^TBx \leqslant \lambda_{\max}|x|^2. \end{cases}$$

Theorem 3. *Suppose*

i) $t - p(t) \to \infty$, $p(t) \to \infty$ *as* $t \to \infty$;

ii) $\|B_2\| + \int_0^{+\infty} \|C_2(t)\|\,dt < 1$;

iii) $1 - 2\|B\|\dfrac{\lambda_{\max}}{\lambda_{\min}}\left[\|B_1\| + k\|B_2\| + \int_0^{+\infty} \|C_1^1(t)\|\,dt + K_1\int_0^{+\infty} \|C_2^1(t)\|\,dt\right] \geqslant 0$;

where

$$k=\frac{\|A\|+\|B_1\|+\int_0^{+\infty}\|C_1(t)\|\,dt}{1-\left[\|B_2\|+\int_0^{+\infty}\|C_2(t)\|\,dt\right]}$$

then the zero solution of (5) *is asymptotically stable.*

Proof. Let

$$k_1(\sigma)=\left[\|A\|+\int_0^{+\infty}\|C_1(u)\|\,du+\|B_1\|\right]\sigma$$

$$k_2(\sigma)=\left[\|B_2\|+\int_0^{+\infty}\|C_2(u)\|\,du\right]\sigma$$

$$k(\sigma)=K\sigma$$

$$q_i(t)=\int_{t-p(t)}^{+\infty}\|C_i(u)\|\,du \qquad (i=1,\,2).$$

It is easy to prove Conditions 1 and 2 hold for the k_1, k_2, k defined above.

$$\left|Ax(t)+B_1x(p_1(t))+B_2x(p_2(t))+\int_{-\infty}^{t}C_1(t-s)x(s)\,ds+\int_{-\infty}^{t}C_2(t-s)\dot{x}(s)\,ds\right|$$
$$\leqslant k_1\big(\sup_{p(t)\leqslant s\leqslant t}|x(s)|\big)+k_2\big(\sup_{p(t)\leqslant s\leqslant t}|\dot{x}(s)|\big)+q_1(t)\|x_t\|+q_2(t)\|\dot{x}_t\|.$$

From Condition (i), it follows that $q_1(t)\to 0$ as $t\to\infty$. So, the Condition 3 hold.

Let $V(t,x)=x^TBx$, $U(|x|)=\lambda_{\min}|x|^2$, $W(|x|)=\lambda_{\max}|x|^2$ then

$$\dot{V}_{(5)}(x(t))\leqslant -x^T(t)x(t)+2x^T(t)B\left[B_1x(p(t))+B_2\dot{x}(p_2(t))+\int_{-\infty}^{+}C_1(t-s)x(s)ds\right.$$
$$\left.+\int_{-\infty}^{t}C_2(t-s)\dot{x}(s)\,ds\right].$$

If $V(x(s))\leqslant V(x(t))$

$$|\dot{x}(s)|\leqslant k\left(\sqrt{\frac{V(x(t))}{\lambda_{\min}}}\right)\leqslant K\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}|x(t)|$$

for $s\leqslant t$, then

$$|x(s)|\leqslant\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}|x(t)|$$

$$\dot{V}_{(5)}(x(t))$$
$$\leqslant -|x|^2\left[1-2\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}\|B\|\left(\|B_1\|+K\|B_2\|+\int_0^{+\infty}\|C_1(u)\|\,du+\int_0^{+\infty}\|C_2(u)\|\,du\,K\right)\right]$$
$$\leqslant 0.$$

Thus, the zero solution of (5) is stable.

If for $p(t)\leqslant s\leqslant t$, $V(x(s))\leqslant N(t)$, $|\dot{x}(s)|\leqslant k(\sqrt{N(t)/\lambda_{\min}})$, then

$$|\dot{x}(s)|\leqslant K\sqrt{\frac{N(t)}{\lambda_{\min}}}\qquad p(t)\leqslant s\leqslant t$$

$$|x(s)|\leqslant\sqrt{\frac{N(t)}{\lambda_{\min}}}\qquad p(t)\leqslant s\leqslant t$$

$\dot{V}_{(5)}(x(t))\leqslant F(t, V, N)$

$$F(t, V, N)$$
$$=-\frac{V}{\lambda_{\max}}+\frac{2\|B\|N}{\lambda_{\min}}\left(\|B_1\|+K\|B_2\|+\int_0^{+\infty}\|C_1(u)\|du+K\int_0^{+\infty}\|C_2(u)\|du\right)$$
$$+2g_2(t)M\|B\|$$

where $|x(t)|^2\leqslant M$; $|\dot{x}(t)|^2\leqslant M$

$$g_2(t)=\int_{t-p(t)}^{+\infty}[\|C_1(u)\|+\|C_2(u)\|]du\longrightarrow 0\qquad\text{as } t\to\infty$$

$$F(t, V, V)$$
$$=-\left[\frac{1}{\lambda_{\max}}-\frac{2\|B\|}{\lambda_{\min}}\left(\|B_1\|+K\|B_2\|+\int_0^{+\infty}(\|C_1(u)\|+K\|C_2(u)\|)du\right)\right]V$$
$$+2g_2(t)M\|B\|$$

$$|F(t, V, N_1)-F(t, V, N_2)|$$
$$\leqslant\frac{2\|B\|}{\lambda_{\min}}\left[\|B_1\|+K\|B_2\|+\int_0^{+\infty}(\|C_1(u)\|+K\|C_2(u)\|)du\right]|N_1-N_2|.$$

From Theorem 2, we get the asymptotical stability of the zero solution of (5).

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